Supplemental Material for "Universal Scaling Laws for Dense Particle Suspensions in Turbulent Wall-Bounded Flows"

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DERIVATION OF DRAG LAW

We derive a relation for the frictional drag, expressed in terms of a friction Reynolds number, Re_τ, from the scaling considerations presented in the Letter. The nomenclature is consistent with the one used in the Letter. In the single-phase case, one can relate the bulk velocity to the friction velocity from the logarithmic scaling laws for mean velocity and velocity defect. A detailed derivation, together with the inherent assumptions can be found e.g. in [S. B. Pope *Turbulent flows.* Cambridge university press, 2000].

We aim at relating Re_τ to the parameters governing the flow in the overlap region: Re_b, Φ and D_p/h. As for single-phase turbulent channel flow we assume for the homogeneous suspension region that the bulk velocity is well approximated by integrating the velocity defect over the height of the homogeneous suspension region (HSR). This approximation is valid as long as (i) the Reynolds number is sufficiently high that the inner layer of the HSR does not contribute significantly to the bulk velocity and (ii) the virtual wall origin δ_{pw} is sufficiently small that the flow inside the particle-wall layer contributes little to the bulk velocity. Thus,

$$U_b \approx \frac{1}{h-\delta_{pw}}\left(\int_{\delta_{pw}}^{h} u\,\mathrm{d}y\right). \tag{1}$$

The bulk velocity is then estimated by integrating the defect law from δ_{pw} to h (consistency requires that the constant B_d, typically small, is set to 0),

$$U_b \approx \left(U_c - \frac{u_\tau^*}{\kappa}\right). \tag{2}$$

Next, the two expressions for the log law, in inner and outer variables respectively, are combined to relate the mean centerline velocity U_c to the apparent wall friction velocity u_τ^*, yielding:

$$\frac{U_c}{u_\tau^*} = \frac{1}{\kappa}\ln\left(\frac{h-\delta_{pw}}{\delta_v^{e*}}\right) + B + B_d \tag{3}$$

Combining Eqs. (2) and (3) we obtain the following expression for U_b/u_τ^*:

$$\frac{U_b}{u_\tau^*} = \frac{1}{\kappa}\left[\ln\left(\frac{h-\delta_{pw}}{\delta_v^{e*}}\right) - 1\right] + B + B_d \tag{4}$$

Substituting $u_\tau^* = u_\tau(1-\delta_{pw}/h)^{1/2}$, and $\delta_v^{e*} = \nu_e/u_\tau^*$ in Eq. (4) we get

$$\frac{U_b}{u_\tau} = \left(\frac{1}{\kappa}\left[\ln\left(\mathrm{Re}_\tau\frac{\nu}{\nu^e}\left(1-\frac{\delta_{pw}}{h}\right)^{3/2}\right) - 1\right] + B + B_d\right)\left(1-\frac{\delta_{pw}}{h}\right)^{1/2} \tag{5}$$

After re-arranging, we finally obtain

$$\mathrm{Re}_\tau = \frac{\mathrm{Re}_b}{2\xi_{pw}^{1/2}}\left(\frac{1}{\kappa}\left[\ln\left(\mathrm{Re}_\tau\chi^e\xi_{pw}^{3/2}\right) - 1\right] + B + B_d\right)^{-1}, \tag{6}$$

where $\xi_{pw} = (1-\delta_{pw}/h)$ and $\chi^e = \nu/\nu^e$. Eq. (6) can be solved numerically by substituting $\delta_{pw} = C(\Phi/\Phi_{max})^{1/3}D_p$ and $\nu_e = (1+(5/4)\Phi/(1-\Phi/\Phi_{max}))^2\nu$. The constant $C = O(1)$ was set to 1.5 for all the cases presented in this study, and Φ_{max} to 0.6.

AN ALTERNATIVE CORRELATION FOR THE OVERALL DRAG

Fig. S1 displays the same quantity as Fig. 4 of the manuscript: the relative difference between predicted

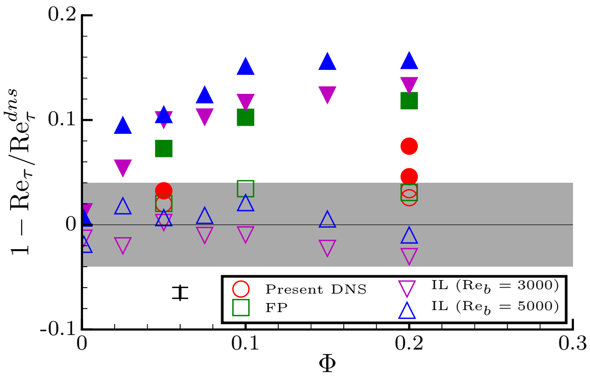

FIG. S1. Relative difference to the theoretical prediction of friction Reynolds number (the shaded area corresponds to a difference of $\pm 4\%$) when an empirical correlation is used: $\mathrm{Re}_\tau = 0.09(\mathrm{Re}_b \chi^e \xi_{pw})^{0.88}/(\xi^{3/2}\chi^e)$. Filled symbols correspond to values that were not for corrected for the presence of the particle-wall layer (i.e. $\delta_{pw} = 0$).The maximum statistical error in the computation of the overall drag (with 95% confidence interval) from the DNS is below 1%. The corresponding (shifted) error bar is also shown on the left hand side of the plot legend.

values of Re_τ and the values obtained from the DNS, Re_τ^{dns}. The difference now is that the estimate is based on an empirical correlation valid for single-phase flow ($\mathrm{Re}_\tau^{sph} \approx 0.09\mathrm{Re}_b^{0.88}$ [S. B. Pope *Turbulent flows*. Cambridge university press, 2000]), which is extended to the case of a turbulent suspension. For the homogeneous suspension region (see the modeling considerations in the Letter) we obtain:

$$\mathrm{Re}_\tau^{hsr} = \frac{u_\tau^*(h-\delta_{pw})}{\nu^e} \approx 0.09\left(\frac{U_b(h-\delta_{pw})}{\nu^e}\right)^{0.88}, \quad (7)$$

and from this we derive the following explicit, power-law expression for Re_τ:

$$\mathrm{Re}_\tau = \frac{0.09\,(\mathrm{Re}_b \chi^e \xi_{pw})^{0.88}}{\xi_{pw}^{3/2}\chi^e}; \quad (8)$$

where $\xi_{pw} = (1 - \delta_{pw}/h)$ and $\chi^e = \nu/\nu^e$. Fig. S1 shows that the empirical correlation given by Eq. (8) yields similar predictions for the drag as Eq. (3) in the Letter. In general, the predictions from the empirical correlation are slightly more accurate (i.e., the error is smaller), in particular for the data at the lowest values of Re^{hsr} (see upward- and downward-pointing triangles).

It is interesting to note that the explicit nature of Eq. (8) enables us to estimate the relative importance of the finite-size effect (ξ_{pw}) and effective suspension viscosity (χ^e) at the given flow rate (quantified by Re_b):

$$\mathrm{Re}_\tau \propto \frac{\mathrm{Re}_b^{0.88}}{\xi_{pw}^{0.62}\chi^{e\,0.12}}. \quad (9)$$

The large exponent of ξ_{pw}, 0.62, confirms that the finite-size effect plays an important role. At fixed particle size, however, the effective viscosity still plays a major role in these dense flows, as $1/\chi^e \sim 1 + \Phi + O(\Phi^2)$ increases faster with Φ than $1/\xi_{pw} \sim 1 + \Phi^{1/3} + O(\Phi^{2/3})$.

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