Self-similar boundary layer lab
– FPG –

FLUID MECHANICS/STRÖMNINGSMEKANIK SG2214

Lab exercise location:	**Strömningsfysiklaboratoriet**
	Teknikringen 8, hall 49, ground floor
Lab exercise duration:	approximately **2h**
Own material:	bring a notepad and a pencil since you are expected to take notes for the lab report

Comments on this Lab-PM and the following appendix can be e-mailed to:
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Prerequisities

Before attending the lab, each student is obligated to ask one unique question on a subject related to the lab PM (theory, procedure, instructions etc.). A student without a prepared question will not be allowed to perform the lab. Note that the lab will be performed in groups of four, implying that you need to prepare at least four questions in order to ensure that you have a unique question to ask. Time for questions is given during the brief introduction of the lab, which is given by the assistant.

Aims and objectives

The aim of the present lab is:

- to deepen your understanding of self similarity

- to develop your knowledge of the effect of pressure gradients on boundary layers

- to get acquainted with the performance and evaluation of measurements

Examination

In this course (SG2214) the lab and lab report is part of the homework assignments, INL1 (3 ETC). The lab including the report roughly corresponds to a three-days work load. The lab exercise involves data acquisition (about 2 hours), data analyses, derivation of the von Kármán momentum integral equation and comparison with theoretical/numerical results from the Falkner-Skan similarity solution. A passing grade for the homework assignment will be awarded when your lab report is approved. See detailed instructions about the analyses and content of the report in the appendix of this lab PM.
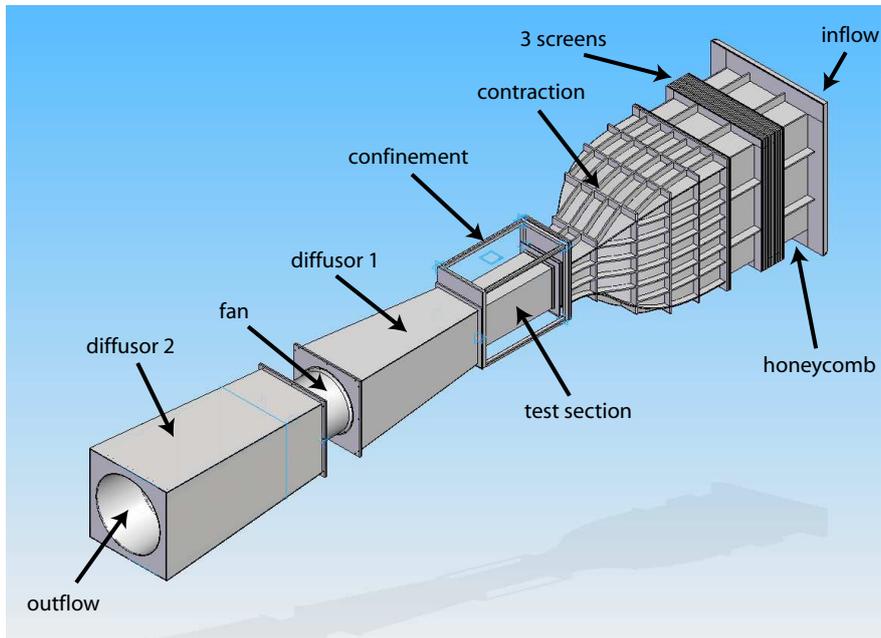
Figure 1: A sketch of the open-circuit wind tunnel.

Experimental setup

The lab will be performed in a open-circuit wind tunnel and in figure 1 a sketch of this circuit is shown. The flow is driven by an axial fan (frequency controlled AC-motor), which is located downstream of the test section. As may be noted, the test section of the wind tunnel constitutes only a fraction of the entire circuit and in figure 2 a close-up of the test section is shown. The cross-sectional area of the test section is 0.40×0.50 m^2 (width $\times$ height). In order to obtain a good flow quality in the test section, i.e. low levels of velocity fluctuations and a uniform velocity profile, a honeycomb followed by three successively finer meshed screens are installed at the inflow of the wind tunnel. The honeycomb directs the flow to become parallel to the centre line of the test section, and the screens make sure to successively break down the eddies in the flow. The contraction directly upstream of the test section also provides a strong damping of relative fluctuation levels and velocity variations in the cross-sectional area of the test section.

The setup, shown in figure 2, consists of a flat plate with pressure holes at different downstream locations from the leading edge in order to allow for measurements of the static pressure, a traversable total pressure tube probe and an exchangeable bump, which can be mounted in the ceiling in order to setup a boundary layer edge velocity variation close to $U_e(x) = ax^n$. This variation is taken as an *ansatz* in deriving the Falkner-Skan similarity equation in the next section. The bump will be mounted in the setup on your arrival so that an accelerating flow is obtained giving rise to a Falkner-Skan boundary layer over the downstream part of the plate. Three experimental measurements of

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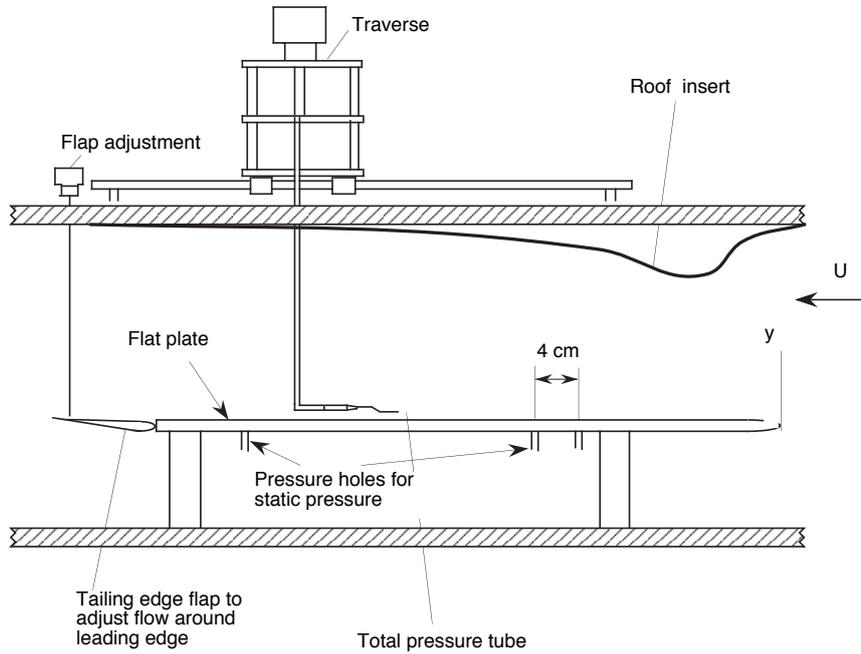
Figure 2: A sketch of the flat plate mounted in the test section.

boundary layer profiles through a boundary layer under an accelerating free stream will be performed. Each profile at a different downstream location. The coordinates are x, y and z for the streamwise, wall-normal and spanwise directions, respectively.

The boundary layer velocity profiles are measured indirectly by a total pressure tube probe. In an irrotational flow, such as in the free stream outside the boundary layer, the total pressure, defined as

$$p_{tot} = p_{stat} + \frac{1}{2}\rho u^2 \; , \qquad (1)$$

is constant (Bernoulli's equation). In equation (1), p_{stat} is the static pressure, ρ the density of the fluid and u the velocity. A boundary layer is not irrotational, which is why the total pressure is not constant. Instead, one can use the fact that the static pressure p_{stat} is constant through the boundary layer in order to determine the velocity at the position of the total pressure probe. The dynamic pressure, defined as

$$p_{dyn} = \frac{1}{2}\rho u^2 \; , \qquad (2)$$

which corresponds to the second term on the right hand side of eq. (1) is obtained by measuring the pressure difference between the static pressure and the total pressure.

The pressure difference between the static pressure on the flat plate and and the total pressure at some distance above the plate is connected to a pressure transducer, which converts the pressure to an analog voltage. The voltage

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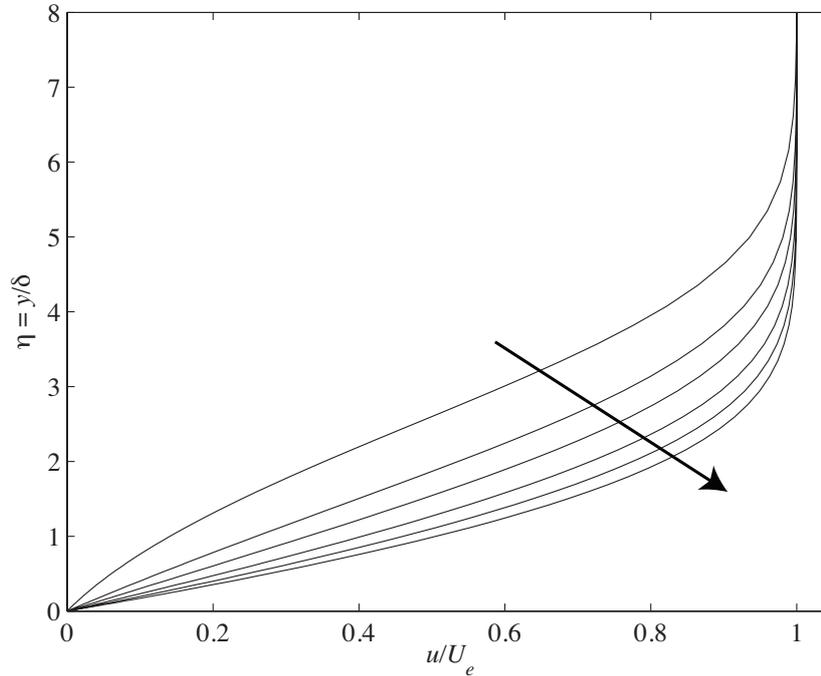
Figure 3: Falkner-Skan solutions to some different n values. The arrow points in the direction of increasing $n = -0.08, \ -0.04, \ 0, \ 0.06, \ 0.12, \ 0.18$.

is measured by an AD-converter and saved on disk by a computer for later evaluation.

The pressure gradient over the plate (or indirectly the free stream velocity distribution) is determined by measuring the pressure at 16 pressure holes in the plate (separated by about 4 cm), by means of an inclined pressure manometer giving the pressure differences in cm meth pillar.

Theoretical background

For a boundary layer, a pressure gradient is the same as a varying free stream velocity, since Bernoulli's law is valid in the free stream. A decrease/increase in the free stream velocity corresponds to an increase/decrease in the pressure. The pressure gradient thus induced is called an Adverse Pressure Gradient (APG), if the pressure gradient is decelerating the flow, and an Favourable Pressure Gradient (FPG), if the pressure gradient is accelerating the flow.

During the lectures the concept of boundary layer similarity will be introduced and the Blasius similarity equation derived. The solution is part of the family of Falkner-Skan similarity solutions, for the specific constraint of zero pressure gradient. The Falkner-Skan similarity equation is derived by opposing a boundary layer edge velocity variation according to

$$U_e(x) = ax^n \ , \tag{3}$$

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which gives rise to a streamwise pressure gradient dp/dx in the x-momentum equation. The Falkner-Skan equation is readily derived as

$$f''' + \frac{n+1}{2}ff'' - nf'^2 + n = 0 \; , \tag{4}$$

where the primes denote derivatives with respect to the similarity variable $\eta = y/\delta$. See the lecture notes for a complete derivation of above equation for the specific case of $n = 0$, i.e. the Blasius equation. Note, that the boundary layer scale δ

$$\delta = \sqrt{x\nu/U_e(x)} \; , \tag{5}$$

is a function of x in both the numerator and the denominator for $n \neq 0$. The equation can be solved numerically using the no-slip boundary condition ($f(0) = f'(0) = 0$) and the free stream condition ($f' = 1$ as $\eta \rightarrow$ *the boundary layer edge*). Having assessed the solution of equation (4), for a given n, the velocity distribution is recovered as

$$\frac{u(x,y)}{U_e(x)} = f'(\eta) \; . \tag{6}$$

In figure 3 the solution of equation (4) is shown for some different n values.

An important quantity when considering the flow past an aerodynamically smooth body is the skin-friction, which is the leading contribution to the overall drag. This quantity has to be assessed in any fuel efficiency estimation when considering any type of aerodynamically smooth vehicle. It is often convenient to express the skin-friction in a dimensionless coefficient, i.e. the skin-friction coefficient (c_f) defined as

$$c_f = \frac{\tau_0}{P_{dyn}} = \frac{2\tau_0}{\rho U_e^2} \; , \tag{7}$$

where P_{dyn} is the dynamic pressure in the free stream and τ_0 is the wall shear stress defined as

$$\tau_0 = \mu \frac{\partial u}{\partial y}\big|_{y=0} \; . \tag{8}$$

Here μ denotes the dynamic viscosity, which has the dimension [kg/(m·s)].

Experimental procedure

The lab will be performed as follows:

- brief introduction to the lab

- modification of the ceiling in order to create a variable free stream velocity close to $U_e(x) = ax^n$

- determination of the streamwise pressure distribution

- experimental measurements of boundary layer profiles through a boundary layer under an accelerating free stream at three streamwise positions

- the experimental data, three velocity profiles and the streamwise pressure distribution, and a folder of useful Matlab programs will be e-mailed to you for post-processing and comparison with the Falkner-Skan similarity solution. Note, a back-up of your data will always be found on one of the lab computers.

Appendix: Lab exercise and report

When you hand in the lab report all pages should be marked with your name and personal civic number. The analytical derivations are to be handed in handwritten and intermediate steps have to be explained in words. For the plotting and analyses some Matlab programming is required. For the Matlab postprocessing of the data a folder, `FPGlab_mfiles`, with Matlab programs will be used. Without this folder you will not be able to carry out the analyses described below. The lab report should contain the following figures and analyses:

(0) you will calculate the air properties at the specific day you carried out the lab.

(1) you will compare the measured velocity profiles with the corresponding solutions to the Falkner-Skan similarity equation. The profiles shall be plotted using the displacement thickness ($\delta_\star$) and the local free stream velocity (or equivalently the boundary layer edge velocity, U_e) to scale the wall-normal coordinate y and the local velocity $u(x, y)$, respectively.

(2) you will use the streamwise pressure distribution to calculate the free stream velocity variation and from that determine the acceleration parameter n.

(3) (i) you will derive the von Kármán momentum integral equation, which correlates the wall shear stress to the boundary layer parameters.
(ii) by using this equation you will express the skin-friction coefficient in an expanded form.

(4) you will calculate the local skin-friction coefficient (c_f) based on the experimental data using the von Kármán momentum integral equation and compare it with the numerical solution.

Below subsections (0–4) describe carefully above four items with step-by-step guidelines on how to carry out the derivations and analyses. Everything that should be included in the lab report are numbered in bold text as **0.1**, **0.2**, ... **4.2** et cetera.

HINT: Start Matlab on the computer and open a new m-file and save it with your name placed in the folder `FPGlab_mfiles`. On the first line you write `clear all` in order to clear all variables each time you run the script. Program the items (0)–(2) and (4) in this m-file for the calculations and plotting (item (3) is only carried out with paper and pencil).

(0)

Air density and viscosity

In the analyses of the experimental data and comparison with the theoretical results you will be needing the dynamic- as well as the kinematic viscosity. The

dynamic viscosity of air with the unit kg/(m·s) is temperature dependent and may be calculated using the Sutherland's law

$$\mu = \mu_0 \left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + S}{T + S} \ , \tag{9}$$

where $\mu_0 = 1.7894 \times 10^{-5}$ kg/(m·s), $T_0 = 273.11$ K and $S = 110.56$ K. The density can be calculated using the universal gas law as follows

$$\rho = \frac{p_{atm}}{R\,T} \ , \tag{10}$$

where p_{atm}, R and T correspond to the atmospheric pressure (Pa), the specific gas constant ($R = 287$ J/(kg·K) for air) and the temperature (K). Having calculated μ and ρ the kinematic viscosity (m^2/s) may be calculated as

$$\nu = \frac{\mu}{\rho} \ . \tag{11}$$

(0.1) μ

(0.2) ρ

(0.3) ν

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(1)

Experimental profiles

In order to analyze the measured mean velocity profiles, taken at the downstream locations $x = 0.23$, 0.41, 0.55 m, you need to read in the data from your data files, and for this you can use the function,

$$\texttt{[Y, Uy] = read\_lab\_data\_JF(file\_name)} \ , \tag{12}$$

provided in the Matlab package. Here, `file_name` is the name of the data file and should be entered as a text string (Example: `file_name = 'fpg_x23_gr1'`). The experimentally measured mean velocity profiles are functions of the wall-normal distance `Y` and are here denoted `Uy`. These profiles are taken at discrete positions above the wall with a known relative displacement, but with an unknown absolute position. This means that the position of the wall, `Ywall`, has to be determined in order to get the absolute positions correct. Once `Ywall` is known the wall position can be subtracted from the y-vector of the profile and in that way re-defining the position of the wall to be at $y = 0$. The Matlab program

$$\texttt{[Ywall, ny] = FPG\_LAB\_JF\_P1(Y, Uy)} \tag{13}$$

takes as argument two vectors, the mean streamwise velocity distribution in the wall-normal direction (`Uy`) and its corresponding wall-normal coordinates (`Y`). The program returns estimates of the wall-position `Ywall` and the acceleration parameter `ny` using the Falkner-Skan similarity equation in an iterative least-squares fit sense to the data. Note the dimensions of the arguments: `Y` [mm] and `Uy` [m/s].

In order to plot the data

$$\frac{u(y/\delta_\star)}{U_e} \tag{14}$$

$\delta_\star$ and U_e have to be determined. The latter can be taken as the mean of the last points of the measured velocity in the free stream (check so that the points chosen to calculate the mean has roughly reached a constant value). The displacement thickness, as defined in the lecture notes, is

$$\delta_\star = \int_0^\infty \left(1 - \frac{u(y)}{U_e}\right) \, \mathrm{d}y \,, \tag{15}$$

which may be integrated using the `TRAPZ` command in Matlab (type "help `TRAPZ`" in the command window and read/learn about `Z = TRAPZ(X,Y)`). **NOTE**: you should add the no-slip boundary condition at the wall before calculating the above integral. The ∞ symbol simply indicates a point *infinitely* far away from the plate, in practice it corresponds to the points in the free stream outside the boundary layer. You should repeat above procedure for each one of the measured profiles.

Theoretical profiles

In order to compare your measured profile with the corresponding Falkner-Skan solution you use the solver

$$\texttt{[f, fp, fpp, fppp, eta] = FS\_solver\_JF(ny)} \,, \tag{16}$$

which takes the only argument `ny` (from program 13) and where `[f, fp, fpp, fppp, eta]` correspond to the similarity function, its derivatives and the similarity variable $[f, \ f', \ f'', \ f''', \ \eta]$. Note, that the similarity variable is $\eta = y/\delta$ and the wanted variable is $y/\delta_\star$. For any solution of the Falkner-Skan equation there is a unique coefficient b relating $\delta_\star$ to δ as $\delta_\star = b\delta$ and hence $y/\delta_\star = \eta/b$. b can be integrated analytically using the theoretical Falkner-Skan solution, provided from program (16), as

$$b = \frac{\delta_\star}{\delta} = \int_0^\infty (1 - f') \, \mathrm{d}\eta = \eta(\infty) - f(\infty) \,. \tag{17}$$

(1.1) the n-values for the three different profiles.

(1.2) two figures showing the unscaled and the scaled data of the three velocity profiles.
Figure 1: the three measured profiles unscaled form, i.e. `Y` (mm) vs `U` (m/s). Add the no-slip condition to the vectors before plotting and add a solid line through the symbols (you can also use different colors and/or different symbols for the three profiles).
Figure 2: the three measured profiles scaled form, i.e. `Y`$/\delta_\star$ vs U$/U_e$. Add the no-slip condition to the vectors before plotting and add the corresponding Falkner-Skan profile (program 16). Take n to be the averaged value from the three profiles using program (13). To the figures you should add the correct labels using the Matlab commands `xlabel` and `ylabel`. NOTE: save the b-value calculated using (17) with η and f values corresponding to the averaged n-value of the three profiles using (16), it will be used later for calculating δ using relation (17) in **(4.1)**.

(2)

Free stream velocity variation

The static pressure distribution measured during the lab can be used to calculate the free stream velocity variation and in turn estimate the acceleration parameter n, independent of the method used in exercise (1) above. We use the fact that the total pressure is constant in the free stream and can consequently calculate the free stream velocity using equation (1) as

$$U_\infty(x) = \sqrt{\frac{2\{p_{tot} - p_{stat}(x)\}}{\rho}} = \sqrt{\frac{2\Delta p}{\rho}} \ . \tag{18}$$

The pressure difference (Δp) above was measured using an inclined pressure manometer where the difference is measured in terms of cm meth pillar (Δh^*) and hence the pressure is recovered as

$$\Delta p = \rho_{meth} \cdot g \Delta h^* \cdot \sin(\beta) \tag{19}$$

where ρ_{meth}, g and β correspond to the density of meth (790 kg/m^3), the gravitational acceleration (9.82 m/s^2) and the angle of inclination of the pressure manometer, respectively. β has been noted down during the lab. Note that when measuring with the inclined pressure manometer you always measure a pressure difference between a pressure of interest (h_{stat}) and a reference pressure (h_{ref}), as an example $\Delta h_{stat} = h_{ref} - h_{stat}$. Thus, in order to retrieve Δp in equation (19) you will be needing Δh_{stat} and Δh_{tot} since

$$\Delta p = \rho_{meth} \cdot g(\Delta h_{stat} - \Delta h_{tot}) \cdot \sin(\beta) = \rho_{meth} \cdot g \Delta h^* \cdot \sin(\beta) \ . \tag{20}$$

Note also that Δh^* in equation (19) should have the dimension [m]. The meth pillar values $h(x)$ in [cm] and the corresponding x-positions can be loaded using the `file_name = 'pressure_data_FPG'` with

$$\text{[X, h] = read\_lab\_data\_JF(file\_name) ,} \tag{21}$$

where the first values correspond to $h_{ref} - h_{stat}(x)$ ($= \Delta h_{stat}$) and the last value in the file to $h_{ref} - h_{tot}$ ($= \Delta h_{tot}$). The corresponding x-positions to $h_{stat}(x)$ is given in X (note that the last element of X is just a dummy zero).

Having calculated the free stream velocity distribution $U_\infty(x)$ (from relation 18), which we here denote `Ux`, you shall use the Matlab program

$$\text{[nx] = Ufit\_JF(X, Ux) ,} \tag{22}$$

which takes as argument two vectors, the mean streamwise velocity distribution in the streamwise direction `Ux` and its corresponding streamwise coordinates `X` [m], where the latter is a priori known and was loaded into Matlab by executing (21). The program (22) returns an estimate of the acceleration parameter `nx` in a least-squares fit sense to the data using relation (3). It plots U versus X along with the curve fitted line. Save the figure and include it in your lab report. Compare the n (`nx`) obtained here with the n-values you obtained using the method in exercise (1) (i.e. `ny`). Bring this in as a result in your lab report.

(2.1) figure of the free stream velocity variation along with the curve fitted line.

(2.2) the n-value, using this method, for the favourable pressure gradient case.

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(3)

Derivation of the von Kármán momentum integral equation

In this exercise you will derive the von Kármán momentum integral equation, which correlates the wall shear stress to the boundary layer parameters. Apart from the displacement thickness eq. (15) we will be using the momentum thickness (θ) defined as

$$\theta = \int_0^\infty \frac{u(y)}{U_e} \left(1 - \frac{u(y)}{U_e} \right) \, \mathrm{d}y \,, \tag{23}$$

see the lecture notes for its derivation.

(i) The von Kármán momentum integral equation is derived by integrating the boundary layer equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{\mathrm{d}U}{\mathrm{d}x} + \nu \frac{\partial^2 u}{\partial y^2} \tag{24}$$

from $y = 0$ to $y = \infty$, where ∞ corresponds to any distance outside the boundary layer. As described in the lecture notes the pressure gradient term (1$^{\mathrm{st}}$ term on the RHS) can be expressed in terms of the velocity $U(x)$ at the edge of the boundary layer, which has been done here. If we add and subtract $u(\mathrm{d}U/\mathrm{d}x)$, we can rewrite eq. (24) as

$$(U - u) \frac{\mathrm{d}U}{\mathrm{d}x} + u \frac{\partial (U - u)}{\partial x} + v \frac{\partial (U - u)}{\partial y} = -\nu \frac{\partial^2 u}{\partial y^2} \,. \tag{25}$$

Now, show by integrating above equation from $y = 0$ to $y = \infty$ that one gets

$$\delta_\star U \frac{\mathrm{d}U}{\mathrm{d}x} + \frac{\mathrm{d}}{\mathrm{d}x}(U^2 \theta) = \frac{\tau_0}{\rho} \,, \tag{26}$$

which is the von Kármán momentum integral equation.

Hints in the integration of equation (25):

- 1$^{\mathrm{st}}$ term on the LHS: use the definition of $\delta_\star$ and the fact that U only depends on x.

- 3$^{\mathrm{rd}}$ term on the LHS: integrate by parts and use the continuity equation, $\nabla \cdot \overline{u} = 0$, to replace the v derivative and the conditions $v = 0$ at $y = 0$ and $u = U$ at $y = \infty$. The result can be combined with the 2$^{\mathrm{nd}}$ term on the LHS to yield

$$\int_0^\infty \frac{\partial}{\partial x} \{u(U - u)\} \, \mathrm{d}y = \frac{\mathrm{d}}{\mathrm{d}x} \int_0^\infty u(U - u) \, \mathrm{d}y \,, \tag{27}$$

show this and use the definition of θ (equation (23) above).

- On the RHS: use the definition of the shear stress equation (8) and the fact that $\nu = \mu/\rho$.

(ii) Show that by expanding equation (26) and using the definition of the skin-friction coefficient (c_f), eq. (7), one can obtain

$$c_f = 2 \left\{ \frac{\mathrm{d}\theta}{\mathrm{d}x} + \frac{1}{U}\frac{\mathrm{d}U}{\mathrm{d}x}(\delta_\star + 2\theta) \right\} . \tag{28}$$

In using above expression to calculate c_f it is necessary to express the x-derivatives with local parameters, specially when using experimental data since the velocity profiles are taken locally in x and hence no derivatives are available.

Show that above derivatives can be expressed as

$$\frac{\mathrm{d}\theta}{\mathrm{d}x} = c^2 \frac{(1-n)}{2}\frac{1}{Re_\theta} \tag{29}$$

$$\frac{1}{U}\frac{\mathrm{d}U}{\mathrm{d}x} = n\left(\frac{c}{Re_\theta}\right)^2\frac{U}{\nu} \tag{30}$$

where we have used the similarity concept and introduced $\theta = c\delta$ in (29), with c being a constant for a given n. Hint 1: use eqs. (5) and (3). Hint 2: the similarity concept also leads to $Re_\theta = c\sqrt{Re_x}$, where Re_θ and Re_x are the Reynolds numbers based on the momentum thickness and the downstream distance, respectively, using the local free stream velocity as the characteristic velocity (note, $Re_\theta = U\theta/\nu$, $Re_x = Ux/\nu$ and $\sqrt{Re_x} = U\delta/\nu$).

(3.1) full derivation of the von Kármán momentum integral equation (26).

(3.2) full derivation of equation (28).

(3.3) full derivation of equations (29) and (30).

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(4)

Skin-friction coefficient comparison between experiment and theory

Here we will be using the indices *theo* and *exp* on the boundary layer parameters in order to indicate if it is the theoretical value or the experimental value we are referring to. Since we are dealing with dimensional quantities we need to have a δ ($= \sqrt{x\nu/U}$) to work with and since there is a pressure bump in the ceiling there will be a virtual origin in the experimental setup, which will not correspond to the position of the leading edge. This means that if we would plug in the actual x-position where we have measured the profile it will not be a fair comparison with theory, and hence we need to take this into account in the comparison. One way of calculating δ directly is to use

$$\delta = \frac{\delta_{\star,exp}}{b} , \tag{31}$$

where $b = \delta_{\star,theo}/\delta$ (see equation 17) is known from exercise (1). Above δ will include any leading edge and bump effects and hence will be used in the rest of this exercise (4). $\delta_{\star,exp}$ in relation (31) is calculated from the experimental data using relation (15). NOTE: In this exercise it is enough if you pick one of the three favourable pressure gradient velocity profiles.

– **Theoretical c_f**

Using equations (6-8) show that c_f can be written as

$$c_f = 2\frac{\nu}{\delta U}f''(0) = \frac{2c}{Re_\theta}f''(0) \; . \tag{32}$$

In calculating the theoretical c_f, denoted `cf_theo`, one would use the intermediate expression in equation (32) using the mean free stream velocity U determined from the experiment in exercise (1), δ from relation (31) and the first element in `fpp` from (16).

Note:
Equation (32) gives c_f based on the velocity gradient at the wall (see eq. 8). Here, the gradient is obtained from the numerical solution to the Falkner-Skan equation, i.e. c_f is calculated without input of any experimentally measured data points close to the wall.

– **Experimental c_f**

The skin-friction coefficient can be calculated using equation (32) by estimating the velocity gradient from data points closest to the wall, but with fairly poor accuracy as soon as the external pressure gradient departures from zero (i.e. when $n \neq 0$). Instead one can calculate c_f from integral boundary layer parameters without knowledge of how the velocity profile looks like close to the wall. In calculating c_f from the experimental data, denoted `cf_exp`, one should use equations (28–30). The procedure is outlined below.

- You have two values of n, from exercise (1) (take the averaged `ny`-value of the three profiles) and (2), and you should calculate the c_f for both of them.

- The c value should be calculated using the experimental momentum thickness, θ_{exp}, using equation (23) with the same procedure as was used to calculate the displacement thickness in equation (15). Note, $c = \theta_{exp}/\delta$, where δ corresponds to the value calculated in equation (31).

- $Re_\theta = U\theta_{exp}/\nu$, where the local free stream velocity U and the kinematic viscosity ν are both known.

- The displacement and momentum thicknesses in equation (28) correspond to the experimental values.

(4.1) c_f from both experiment and theory (2 values originating from the two different n-values and choosing one of the three velocity profiles).

(4.2) quantify the agreement in terms of their ratio (`cf_exp/cf_theo`).