Course Project

Poiseuille Flow

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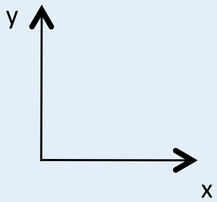
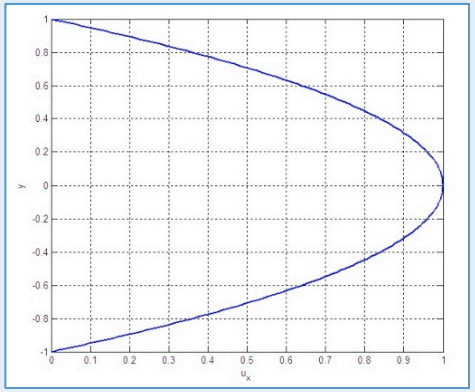




- Navier Stokes equations in x-y dorections govern the flow

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$P = \frac{\partial p}{\partial x} < 0$$

•Boundary conditions:

$$u = 0 \;\; \rightarrow \;\; for\, y = \pm 1$$

•The analitical solution can be found if we impose a parallel flow u(y) and if we consider a steady flow

$$u = \frac{P}{2\rho v} u^2 - \frac{P}{2\rho v}$$

•Disturbance equations are derived from Navier-Stokes equations

- Decomposing velocity and pressure in disturbance and mean value
- Subtracting mean flow
- Linearizing

$$\frac{\partial u_i}{\partial t} + \overline{U}_j \frac{\partial u_i}{\partial x_i} + u \frac{\partial u_i}{\partial y_i} = - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \nabla^2 u_i$$

- Governing stability equations are the
  - Orr-Sommerfeld equation
  - Squire equation

- They can be deriveded from the disturbance equations by assuming parallel flow

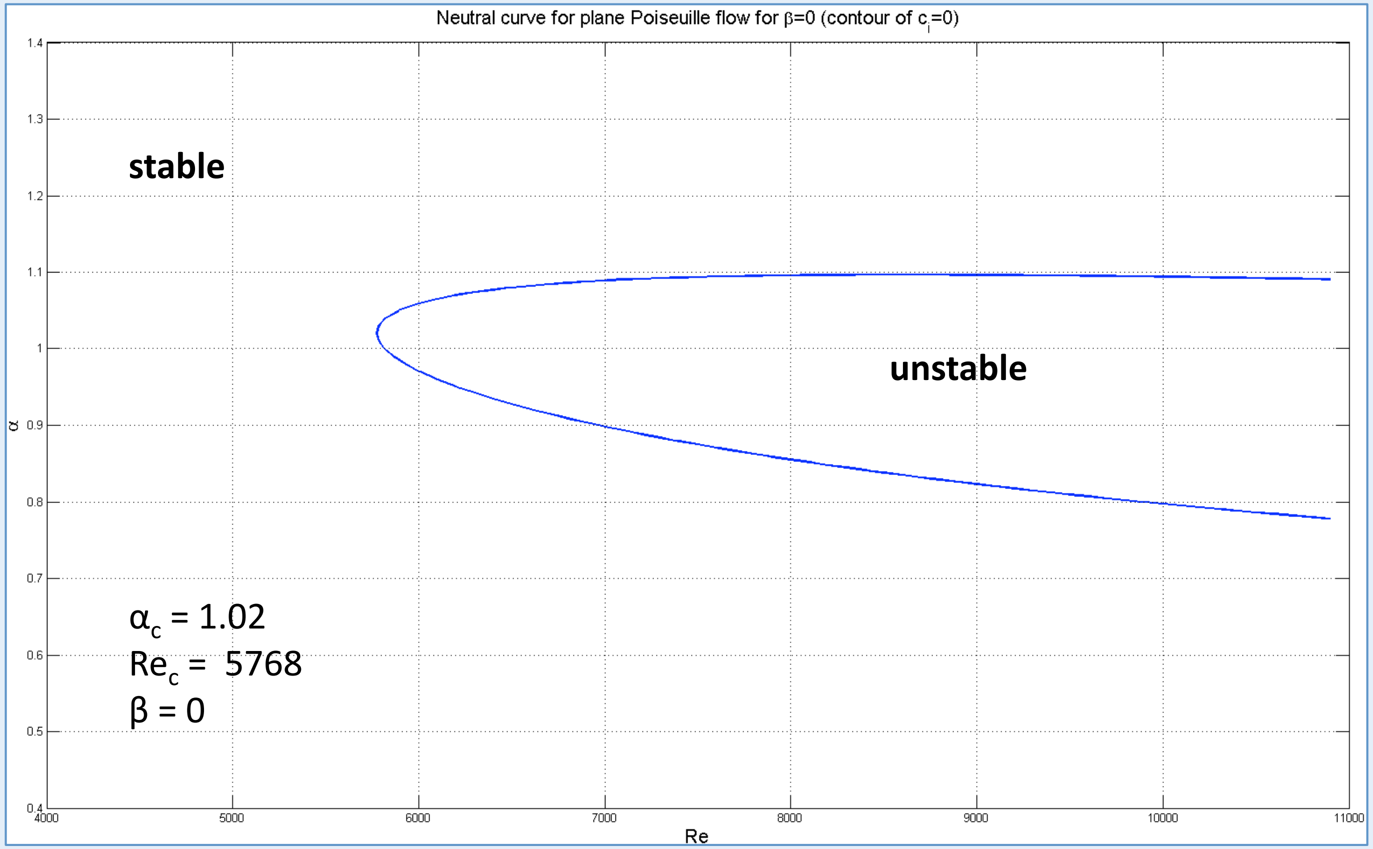
$$\left[\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)\nabla^2-\bar{U}''\frac{\partial}{\partial x}-\frac{1}{Re}\nabla^4\right]v=0$$

$$\left[\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}-\frac{1}{Re}\nabla^2\right]\eta=-\bar{U}'\frac{\partial v}{\partial y}$$

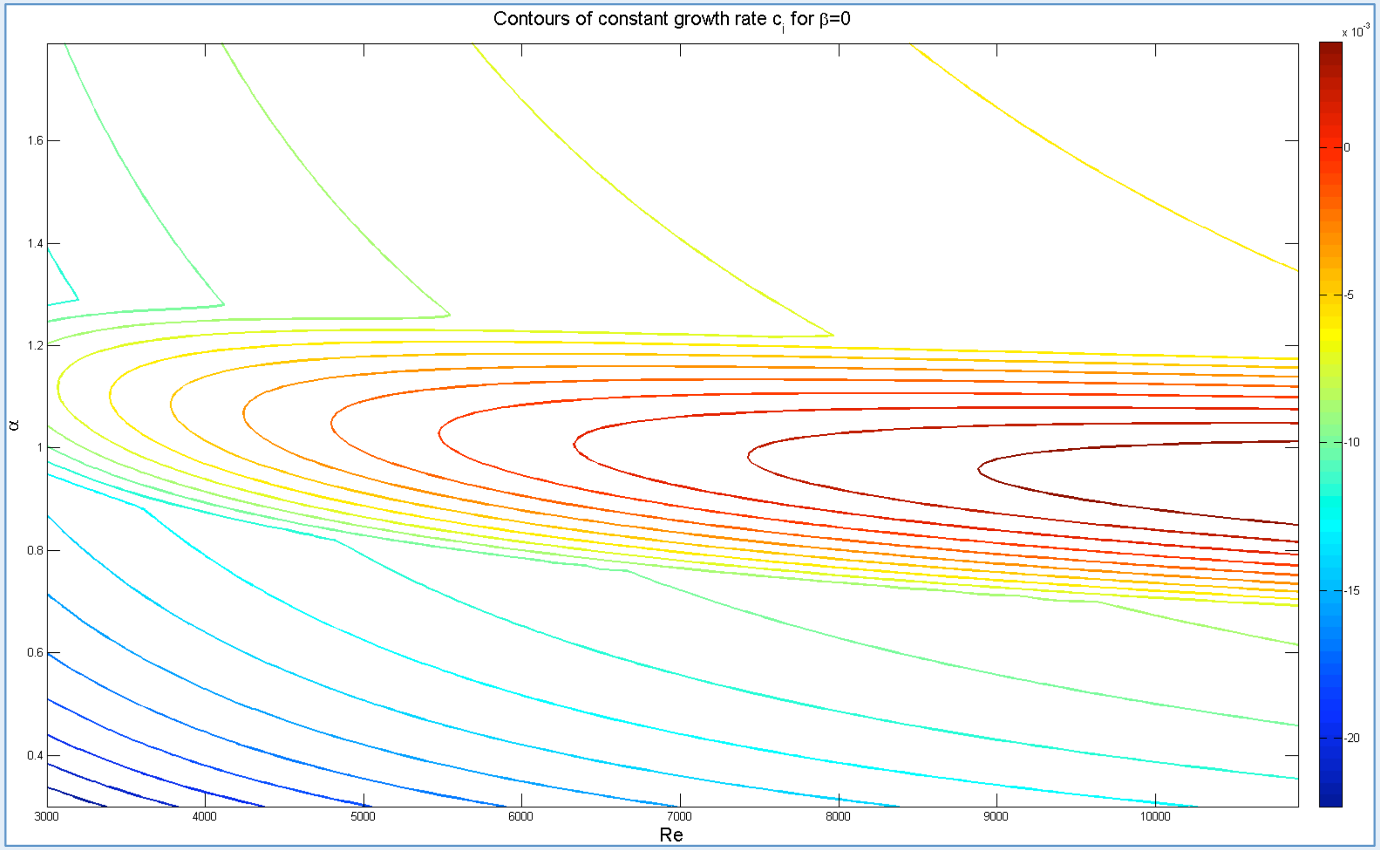
- Boundary conditions

$$v=\frac{\partial v}{\partial y}=\eta=0 \quad \rightarrow for \;\; y=\pm 1$$

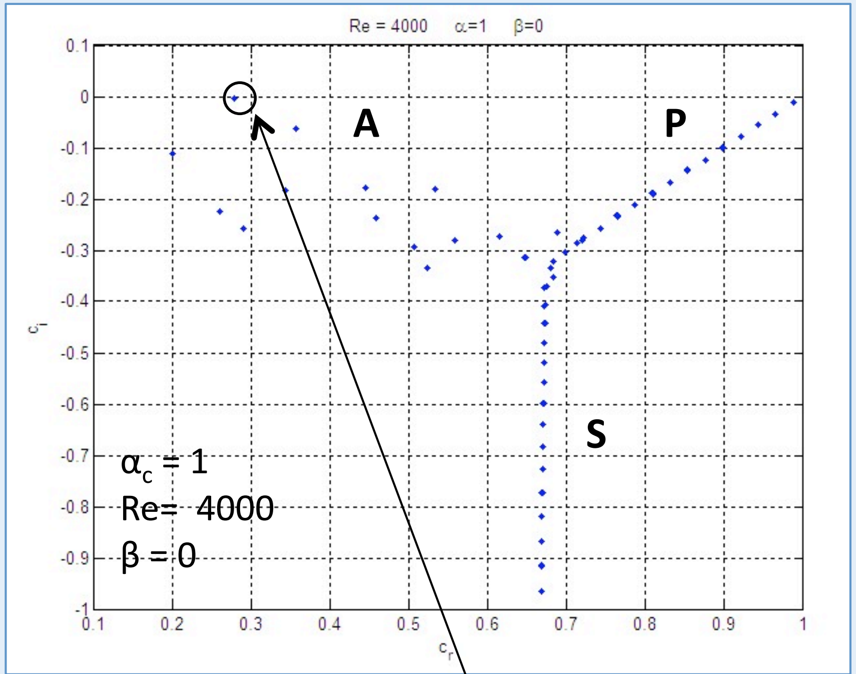
•Neutral stability – Neutral curve

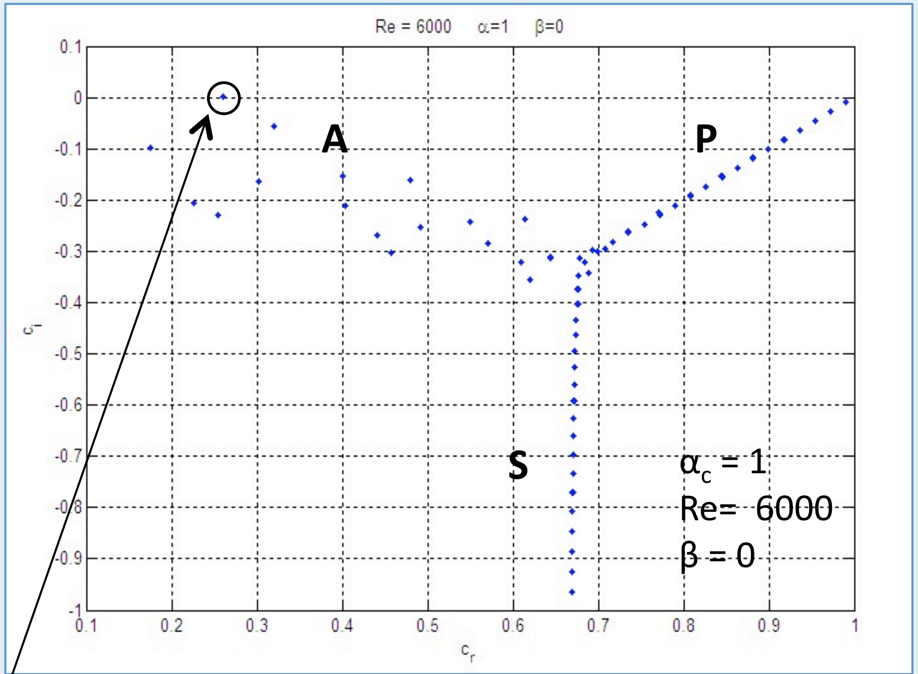


•Neutral stability – Contour of constant growth rate c_i



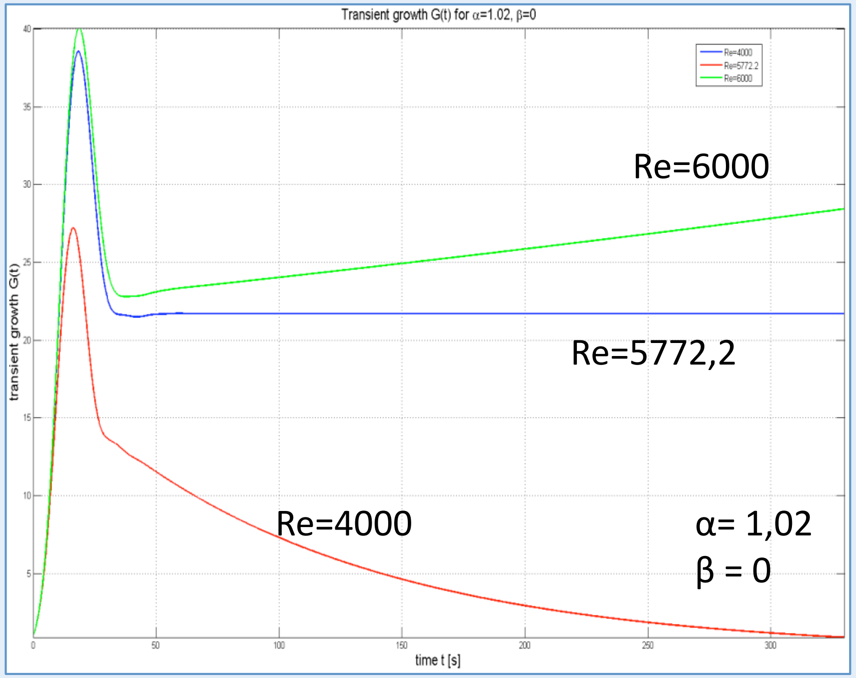
• Spettra stable/unstable cases

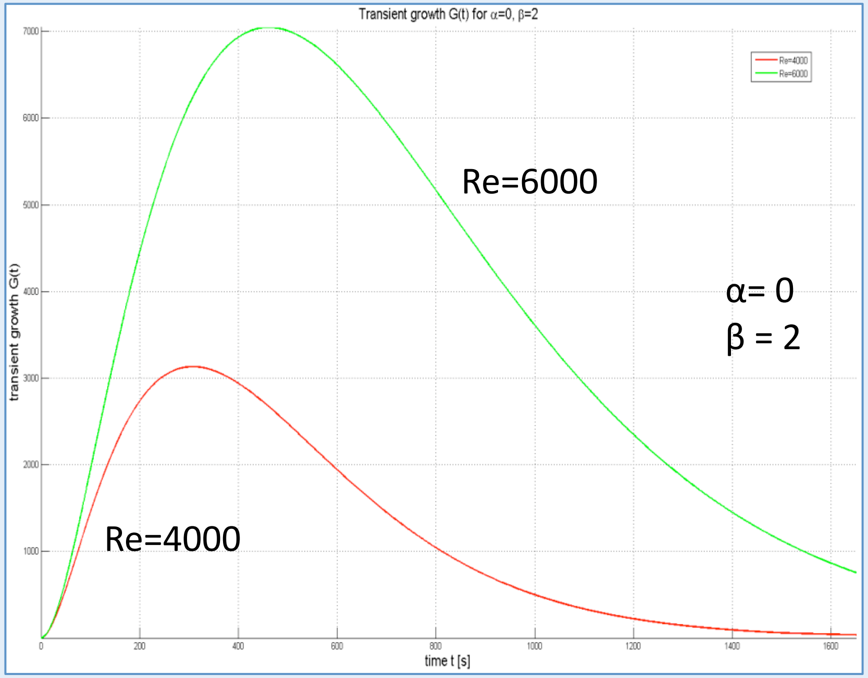




Eigenvalue resposible of instability

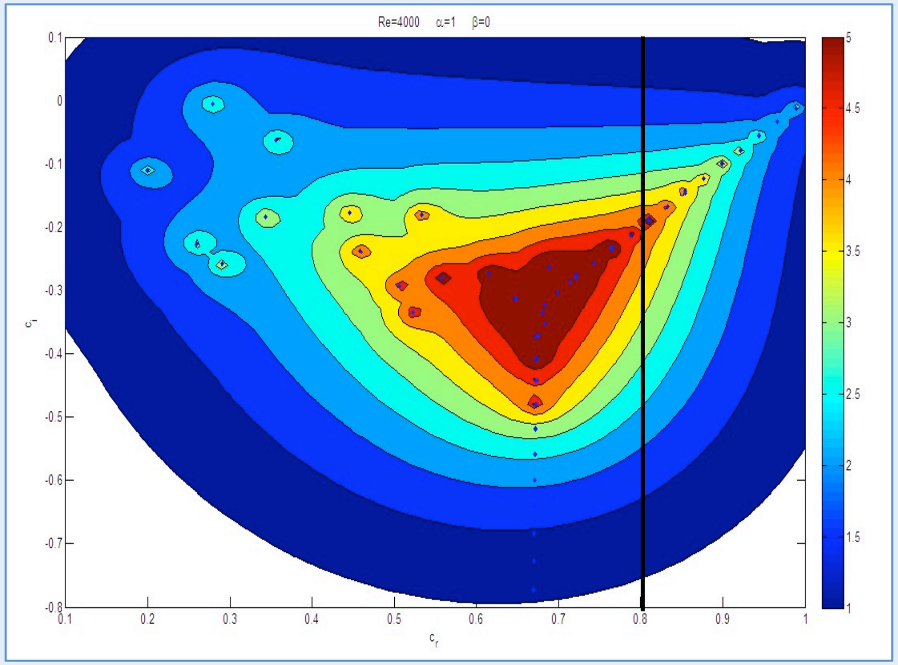
- Transient growth





• Pseudospettra stable/unstable cases

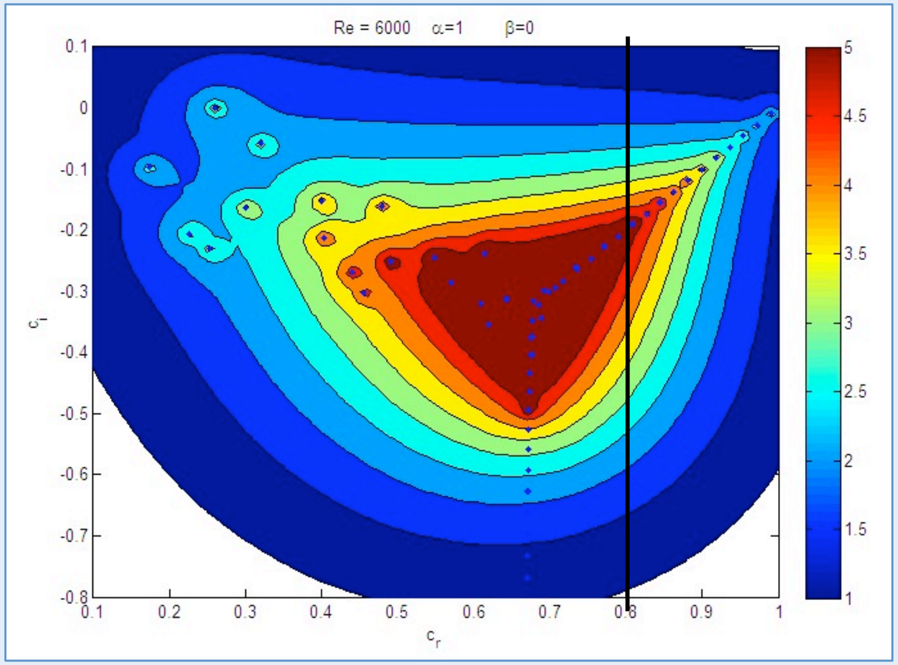
Stable



α = 1
Re= 4000
β = 0

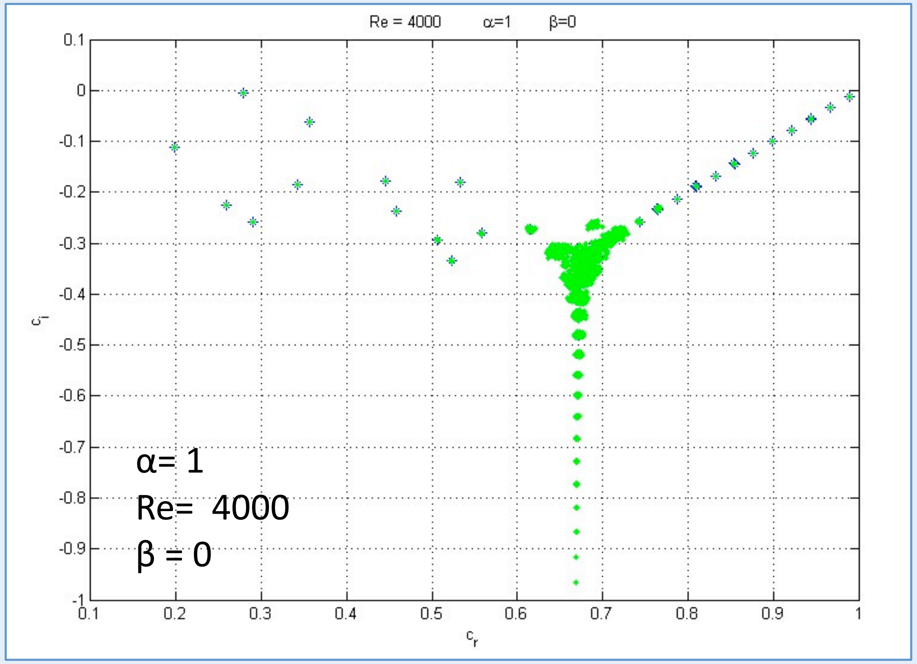
Unstable

α= 1
Re= 6000
β = 0

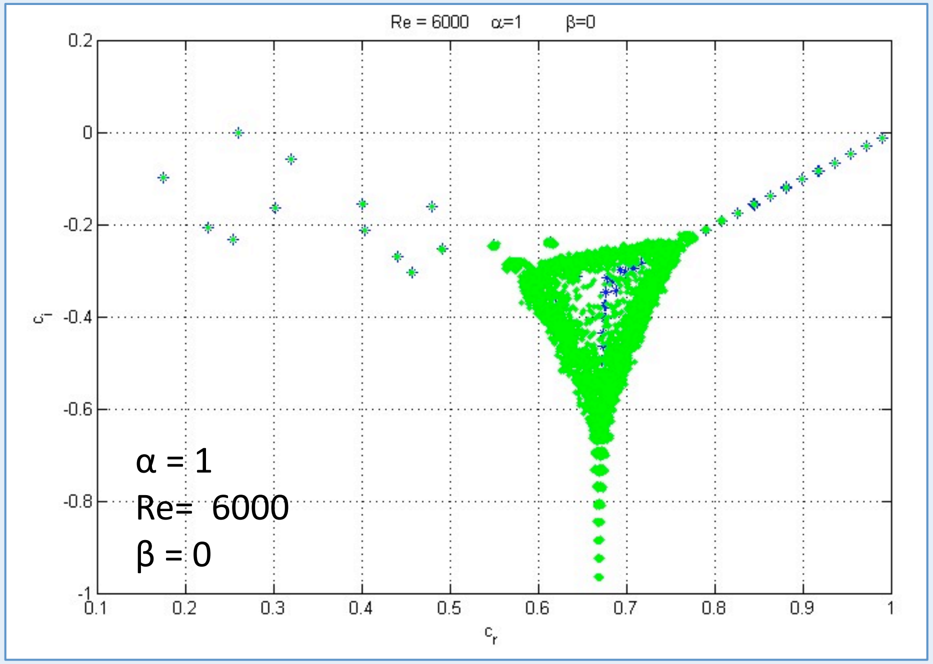


• Sensitivity of eigenvalues stable/unstable cases

Stable

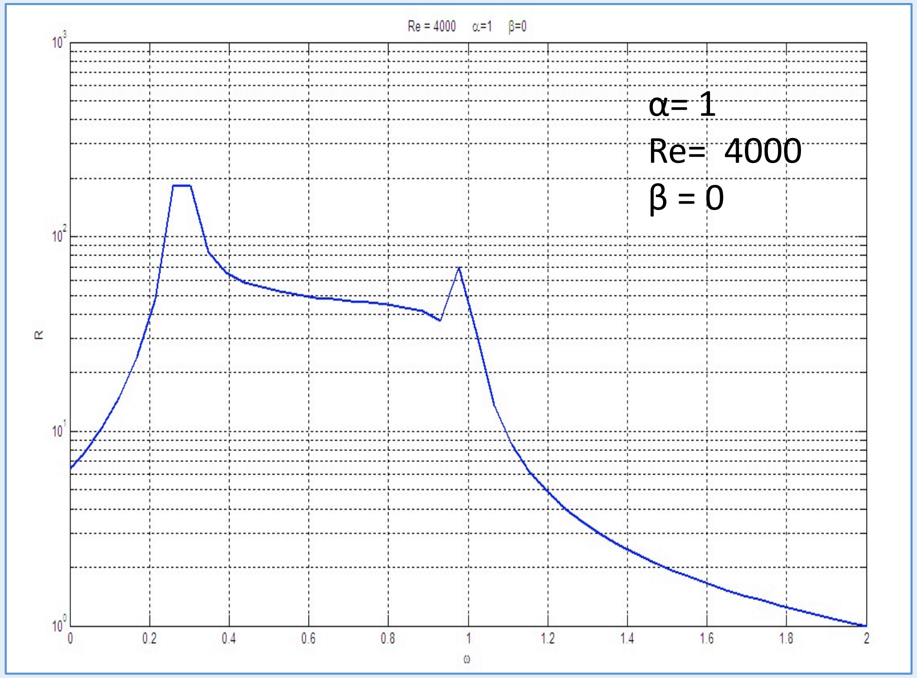


Unstable

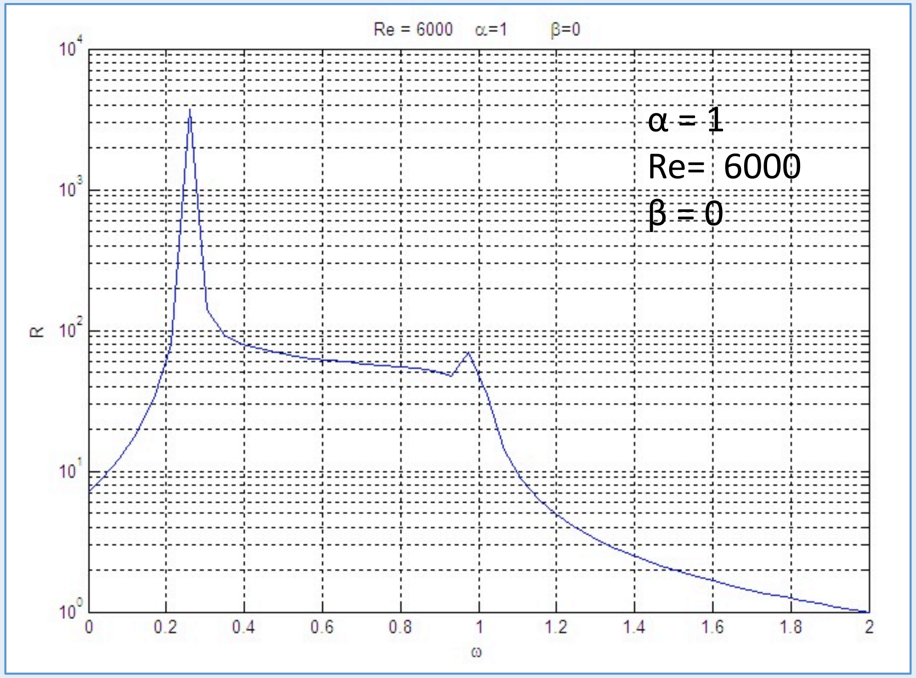


• Optimal responce for stable/unstable cases

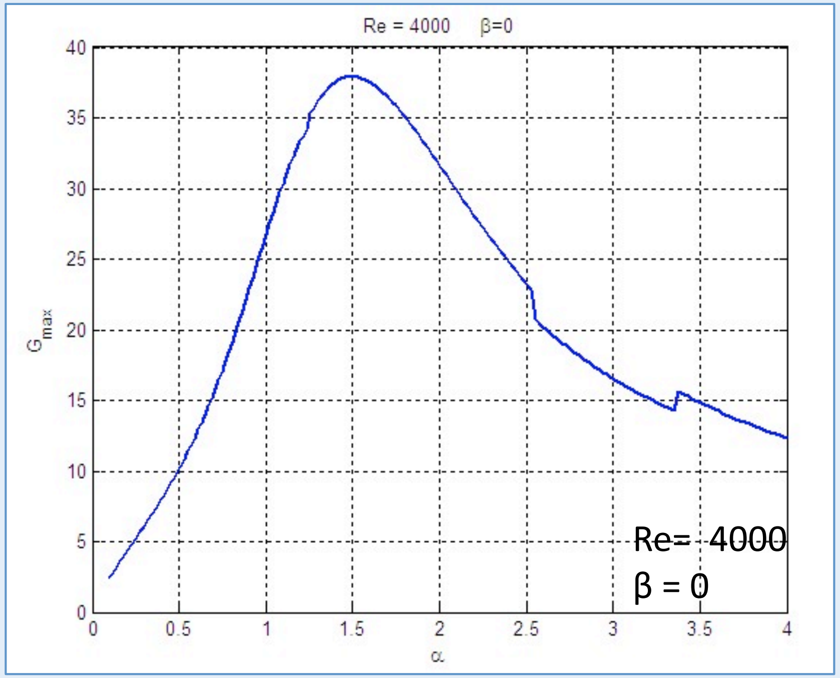
Stable

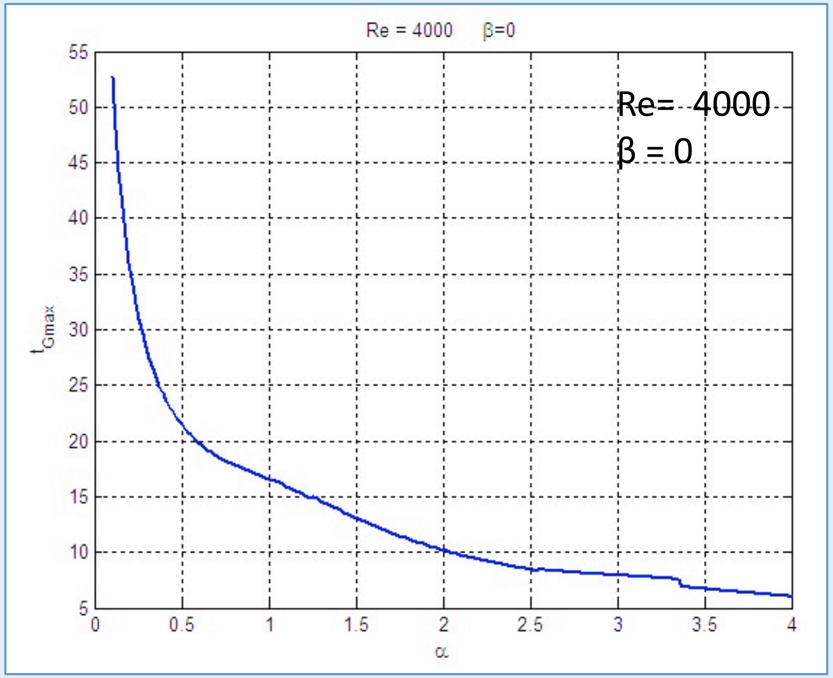


Unstable

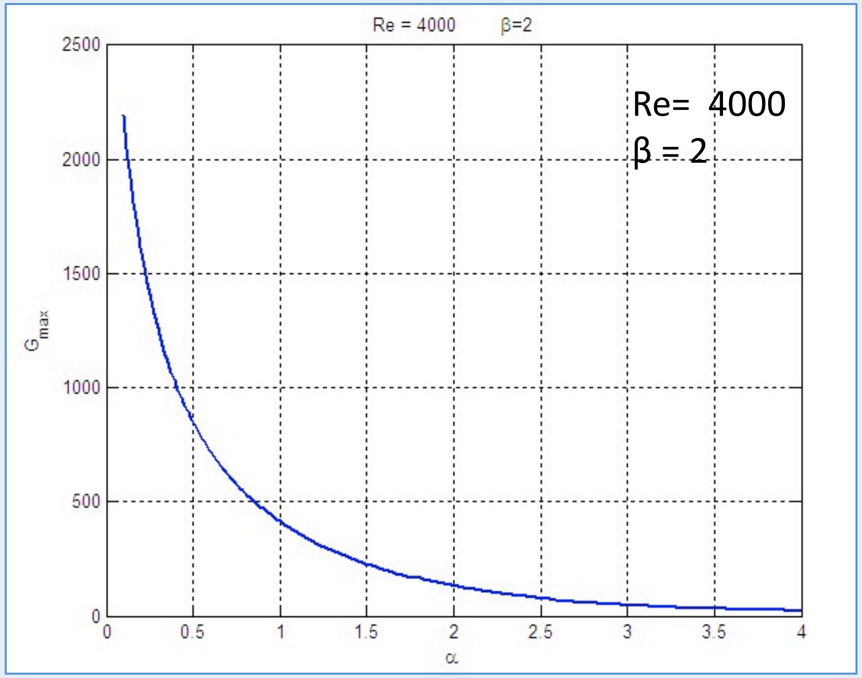


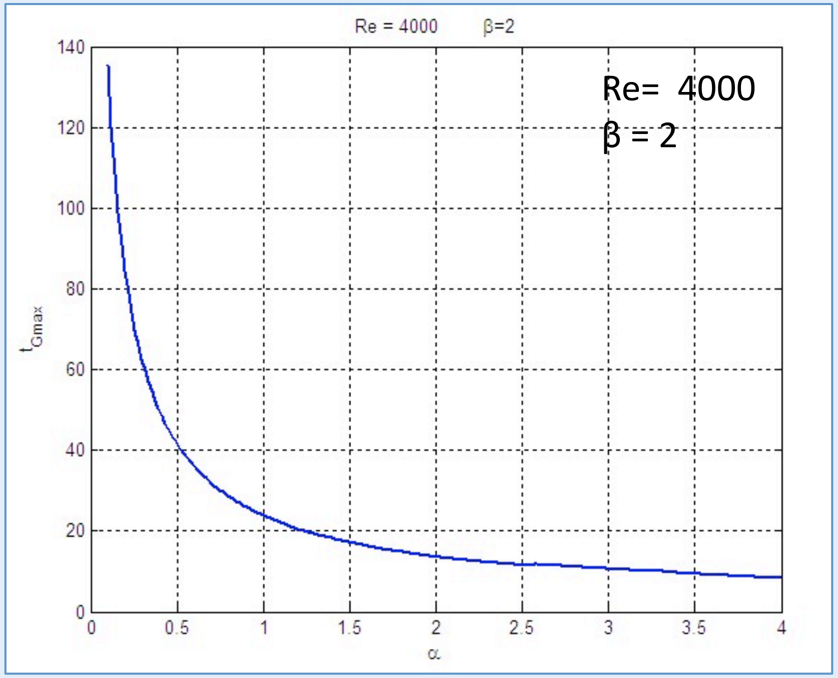
• Maxium transient growth/time against α (β = 0)



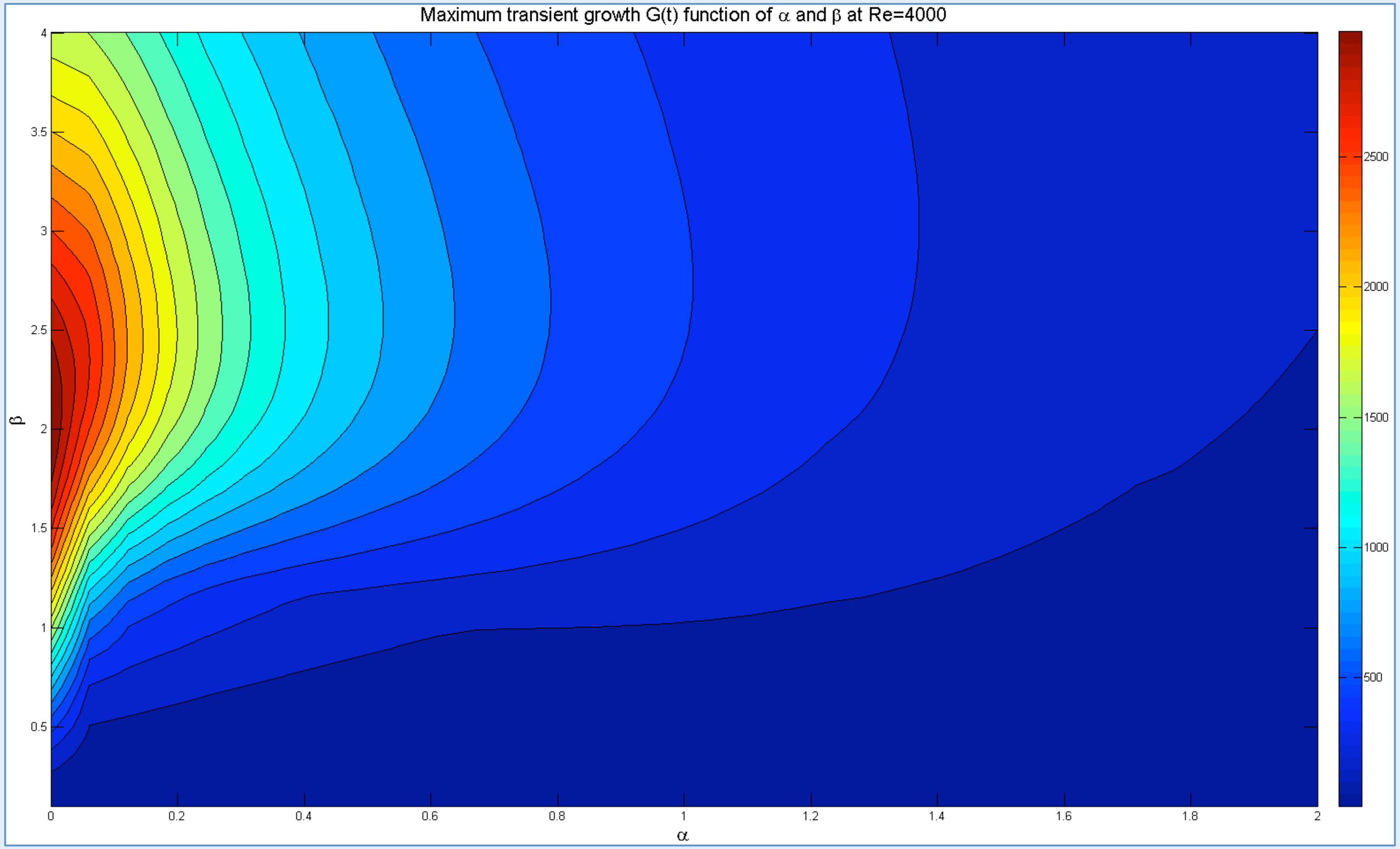


- Maxium transient growth/time against α (β = 2)





- Maxium transient growth in function of α and β



Re= 4000

- Conclusions

- In the inviscid case, the flow is stable since no inflection point is present

- With the modal-analysis the flow results unstable for Re>5772,2 (α=1,02)

- The maximal transient growth occures for α=0 and β=2