High Reynolds number turbulent boundary-layer experiments

Jens M. Österlund and Arne V. Johansson

Department of Mechanics, KTH
SE-100 44 Stockholm, Sweden

Contact e-mail:jens@mech.kth.se

1   Introduction

The flow in a laminar boundary layer is governed by a single length scale and full similarity solutions to the boundary layer equations can be obtained. The case of the turbulent boundary layer flow is quite the opposite with a large span in both velocity and length scales. One can identify two layers with different dynamics, the inner layer where the dynamics are strongly influenced by viscous stresses and an outer layer mainly governed by inertial forces. The velocity and the length scales for the inner layer can be defined from the shear stress at the wall, $u_\tau^2 = \overline{\tau}_w/\rho$ and $\ell^* = \nu/u_\tau$, and for the outer layer as the freestream velocity U_∞ and the thickness of the boundary layer δ. At high Reynolds numbers the ratio of the two lengthscales is large and we have two distinct layers. These meet in an overlap region where the dynamics is essentially independent of lengthscale (an inertial sublayer behavior). Full similarity solutions can be sought for the inner and outer layers independently. The inner layer was treated by Prandtl [6] obtaining solutions of the form

$$\overline{U}^+ = f(y^+), \tag{1}$$

where $\overline{U}^+ = \overline{U}/u_\tau$ and $y^+ = yu_\tau/\nu$. The scaling in the inner layer has since then been verified in numerous experiments. For the outer layer several competing hypotheses for the form exist. The classical form is given by

$$\frac{\overline{U}}{U_\infty} = 1 + \frac{u_\tau}{U_\infty}F(\eta), \tag{2}$$

where $\eta = y/\delta$, see e.g. Österlund [4], ch. 2. For the most recent alternative theories the readers are referred to the works of George [2] and Barenblatt [1] and Zagarola [7].

Millikan [3] obtained the classical logarithmic scaling in the overlap region, by matching the inner and outer layer descriptions Eq. 1 and Eq. 2, here given in inner scaling

$$\overline{U}^+ = \frac{1}{\kappa}\ln y^+ + B. \tag{3}$$

George [2] and Barenblatt [1] use a different approach and arrive at a Reynolds number dependent power-law description of the overlap region although with some differences between the two.

We will here try to identify purely from experimental data, which of the hypothesis that gives the "best" description.

2 Selection criteria

We will test the following hypotheses: a) Re independent log-law, b) Re dependent log-law, c) Re independent power-law and d) Re independent power-law. There are many aspects one could look at in the different approches but the key point of interest must be whether we can identify any significant regions with log- or power-law functional behaviour, Reynolds number dependent or not. This can be done in many ways and the first idea that may spring to mind might be to make lin-log and log-log diagrams of the mean velocity. It turns out to be a rather insensitive test (see Österlund *et. al* [5]). Instead we proceed to look at a normalized gradient of the mean velocity. We define

$$\Xi = \left(y^+ \frac{d\overline{U}^+}{dy^+}\right)^{-1}, \tag{4}$$

that will be constant in a region governed by a log-law, and

$$\Gamma = \frac{y^+}{\overline{U}^+}\frac{d\overline{U}^+}{dy^+} \tag{5}$$

that will be constant in a region governed by a power-law.

To be able to identify any Reynolds number dependence we will look at Eq. 4 and Eq. 5 evaluated for individual measurements of the mean velocity at different Reynolds numbers. One should keep in mind that the scatter necessarily will be quite large when taking the derivative of measurement data.

3 Results

The measurements were carried out at the MTL wind-tunnel at the department of mechanics at KTH and are described in Österlund [4]. The data base for the mean velocity consists of 70 mean velocity profiles taken in the Reynolds number range $2500 < Re_\theta < 27300$. The data are available on the internet at: http://www.mech.kth.se/~jens/zpg/.

Here we will show a subset of the data to make the figures more readable. In figure 1 the gradient of the mean velocity is normalized according to 4 to identify regions with log-law behavior. Above $y^+ \approx 200$ the ensamble of curves flattens out around the constant value 0.38 (performing the same evaluation instead in outer scaling one finds the the outer limit of the overlap at $\eta \approx 0.15$), see Österlund *et. al* [5].

In figure 2 profiles of the mean velocity gradient normalized to identify regions governed by a power-law, are shown. From comparisons of figures 1 and 2 one can conclude that mean velocity profiles closely adhere to a log-law with von Kármán's constant $\kappa = 0.38$. One can also conclude that no significant region with power-law behaviour exists since the function Γ decreases with increasing y^+ in figure 2, for all Reynolds numbers.
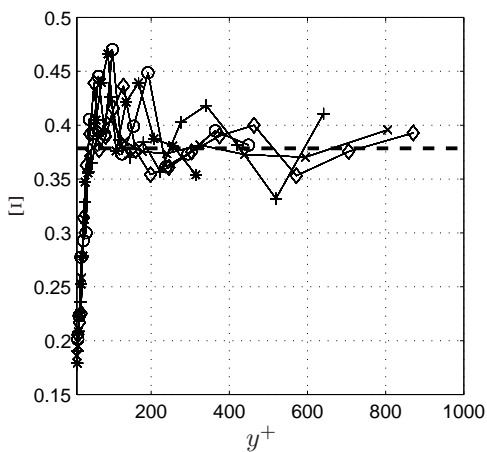

Figure 1: The gradient of the mean velocity normilized to reveal a log-law behaviour. $\eta < 0.15$. Re_θ: $*$: 9600, $\circ$: 14200, $+$:20560, $\diamond$:26600, $\times$:27300. Dashed line: log-law, $\kappa = 0.38$.
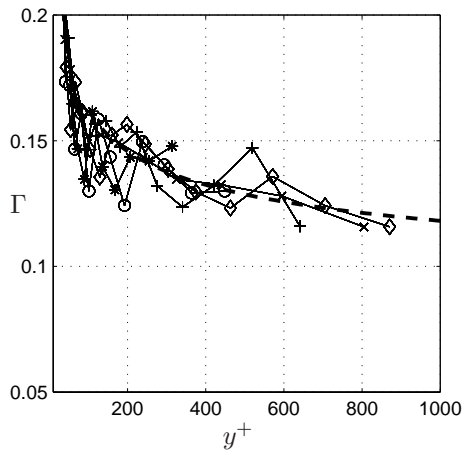
Figure 2: The gradient of the mean velocity normilized to reveal a power-law behaviour. $\eta < 0.15$. Re_θ: $*$: 9600, $\circ$: 14200, $+$:20560, $\diamond$:26600, $\times$:27300. Dashed line: log-law, $\kappa = 0.38$ and $B = 4.1$.

4   Conclusions

- The mean velocity profiles are best described by the log-law in the region $200 < y^+ < 0.15\delta^+$. Hence, a universal overlap region can only be expected to be found for Re_θ larger than about 6000.

- No significant Reynolds number effects were found for sufficiently high Reynolds numbers, yielding the constants $\kappa = 0.38$ and $B = 4.1$ of the log-law, Eq. 3.

References

[1] G. I. Barenblatt, A. J. Chorin, and V. M. Prostokishin. Analysis of experimental investigations of self-similar intermediate structures in zero-pressure boundary layers at large reynolds numbers. PAM 777, Center for Pure and Applied Mathematics, University of California at Berkeley, 2000.

[2] W. K. George, L. Castilio, and M. Wosnik. Zero-pressure-gradient turbulent boundary layer. *Applied Mech. Reviews*, 50:689–729, 1997.

[3] C. B. Millikan. A critical discussion of turbulent flows in channels and circular tubes. In *Proceedings of the Fifth International Congress of applied Mechanics*, 1938.

[4] J. M. Österlund. *Experimental Studies of Zero Pressure-Gradient Turbulent Boundary-Layer Flow*. PhD thesis, Department of Mechanics, Royal Institute of Technology, Stockholm, 1999.

[5] J. M. Österlund, A. V. Johansson, H. M. Nagib, and M. H. Hites. A note on the overlap region in turbulent boundary layers. *Phys. Fluids*, 12:1–4, 2000.

[6] L. Prandtl. Zur turbulenten Strömung in Rohren und längs Platten. *Ergebn. Aerodyn. Versuchsanst. Göttingen*, 4:18–29, 1932.

[7] M. V. Zagarola and A. J. Smits. A new mean velocity scaling for turbulent boundary layers. In *Proceedings of FEDSM'98*, 1998.