Homework 1A 5C1212, due Feb. 5, 2007

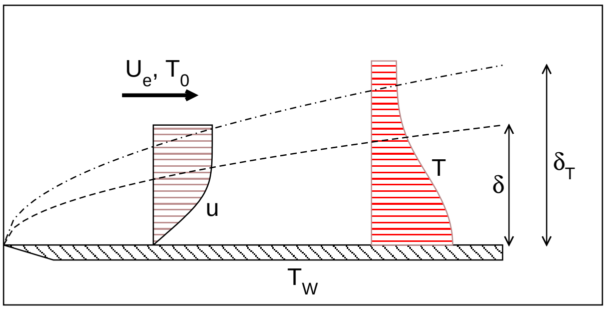
1. An airplane is flying with a trajectory given by $x_i = r_i(t)$. It measures the temperature field $T(t, x_1, x_2, x_3)$ in the surrounding air with a thermometer fixed to the wing. Find an expression for the rate of change of the measured temperature. Explain the physical meaning of the terms and compare the expression with that of the material derivative.

2. Consider the fluid flow between two infinite parallel plates, where the upper plate is moving at a constant velocity V and the lower plate is fixed. Find a steady solution to the incompressible Navier-Stokes equation for this flow case, i.e. so called plane Couette flow.

3. Consider the incompressible flow with free-stream velocity U_e over a semi infinite flat plate. Let the incomming flow have a temperature T_0 while the plate is kept at the greater temperature T_w. The temperature field around the plate obeys the equation

$$\rho\, c_p\, \frac{DT}{Dt} = \kappa \nabla^2 T$$

where κ is the thermal conductivity, ρ density and c_p specific heat at constant pressure. Assume that the Peclet number, $Pe = RePr$, is large so that a thin *thermal* boundary layer develops on the plate.



Derive the appropriate boundary layer approximation of the temperature equation in the limit of large Peclet number and estimate the thermal boundary layer thickness δ_T.

(Hint: assume $\delta/\delta_T \sim O(1)$.)

4. The heat equation for the instantaneous incompressible turbulent flow is

$$\rho\, c_p \left(\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} \right) = \kappa \frac{\partial^2 T}{\partial x_j \partial x_j}.$$

where

$$T = \bar{T} + T', \quad \bar{u}_j = \bar{u}_j + u'_j.$$

Here overbar denotes mean (ensemble averaged) values and prime the perturbations. Derive the heat equation governing the mean flow.