Steady solutions of the Navier-Stokes equations by selective frequency damping

Espen Åkervik, Luca Brandt, Dan S. Henningson,
Jérôme Hœpffner, Olaf Marxen, and Philipp Schlatter*
KTH Mechanics, SE-100 44 Stockholm, Sweden

Abstract. We present a new method that allows the computation of steady solutions of the Navier-Stokes equations in globally unstable configurations. Using this method a steady state is reached by "selective frequency damping" (SFD) of the unstable (temporal) frequencies. This is achieved by employing a dissipative relaxation term proportional to the high-frequency content of the velocity fluctuations. The theoretical formulation of the method is presented and results for cavity-driven boundary-layer separation and a separation bubble induced by a pressure gradient are shown.

The knowledge of a steady base-flow solution of the governing Navier-Stokes equations is fundamental to instability studies and flow control. In the former case it allows for both linear modal and non-modal analyses and weakly nonlinear approaches, whereas in the latter case the stabilization of such a base flow can be adopted as a design target. Unfortunately, when the flow under consideration is globally unstable, it is virtually impossible to numerically compute a steady-state solution of the Navier-Stokes equations by time-marching methods. In a few cases solutions can be obtained by, e.g., enforcing symmetries in the system as in the two-dimensional flow around a circular cylinder. Otherwise, one has to resort to the class of Newton iteration methods, which require heavy computational resources for large systems.

As an alternative, we propose a simple numerical approach to compute steady solutions of the Navier-Stokes equations in unstable configurations [1]. We show that it is possible to reach the steady state by selective damping the most dangerous temporal frequencies and thus quenching the corresponding instability. The method is adapted from large-eddy simulation (LES) techniques, in particular considering the work of Pruett et al. [2] and, using spatial filters, the approximate deconvolution model (ADM) and the ADM-RT model (see [3]).

Consider the general nonlinear system $\dot{q} = f(q)$ (e.g. the Navier-Stokes equations) with appropriate initial and boundary conditions for the vector quantity q under the operator $f(q)$. A steady state q_s is then given by $\dot{q_s} = f(q_s) = 0$. If f is unstable, any $q \neq q_s$ will quickly depart from q_s. In order to stabilize the above system we propose to employ a linear forcing towards a target solution w, i.e. $\dot{q} = f(q) - \chi(q - w)$, with a model coefficient χ. The actual target solution w is taken as a modification of q with *reduced temporal fluctuations*, i.e. a *temporally low-pass filtered* solution $w = T * q$. As q is approaching q_s, the filtered solution $w = T * q$ will in turn approach q, therefore reducing the control influence. If q is the actual steady solution, the time-filtered value w will be identical to $q = q_s$, yielding a vanishing forcing.

For the present cases we consider the differential form of the temporal exponential filter [2], given by the evolution equation $\dot{\overline{q}} = (q - \overline{q})/\Delta$ with an overbar denoting filtered quantities. The complete system now consists of the modified dynamic equation and the filter equation.

The effect of the regularization can be illustrated by considering the eigenvalues of the complete system linearized about the steady state. Assume μ is a complex eigenvalue of the linearized operator f, the eigenvalues related to the modified system λ^+ and those related to

*Corresponding author. Email: pschlatt@mech.kth.se

the filter equation λ^- are obtained as [1]

$$\lambda^\pm = \mu - \chi(1-\alpha^\pm), \text{ with } \alpha^\pm = -\frac{F \pm \sqrt{F^2+4\Delta\chi}}{2\Delta\chi} \text{ and } F := \Delta(\mu-\chi)+1 \; . \tag{1}$$

The mapping $\mu \to \lambda^\pm$ in the complex plane is illustrated in figure 1 for specific parameters χ and Δ. Two lines are represented (indicating possible eigenvalues μ of the original system). Each line is mapped into two curves, the dashed one corresponding to λ^+, and dash-dotted line to λ^-. The arrows indicate how two points of the original solid lines are mapped into the new values corresponding to the new eigenvalues. It can be seen that points with large real part are simply damped. The width of the hump forming at low frequencies is related to the filter cutoff frequency, i.e. $1/\Delta$.

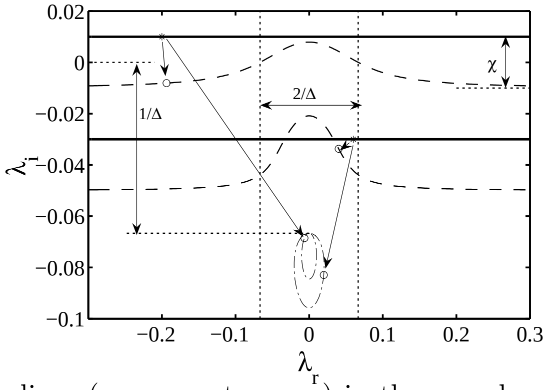

Figure 1: Mapping of two lines (μ_i = const. ——) in the complex plane due to the modified (linear) system. Two points originate from each complex eigenvalue μ, one point corresponding to λ^+ (- - - -) and one corresponding to λ^- (–·–).

To successfully apply the method, the filter cutoff ω_c is related to the frequency of the relevant instabilities and should be smaller than those frequencies at which perturbation growth is expected. The gain χ is related to the growth rates of the instabilities and should be large enough to move the instability modes to the lower half plane. However, choosing a large χ will render the system evolution slow, since the low-frequency eigenvalues associated with the filter, λ^-, move towards the origin of the complex plane. As a guideline, the regularization parameter χ is chosen to be twice the growth rate of the dominant disturbance. The cutoff frequency, $\omega_c = 1/\Delta$ is chosen in such a way that the unstable disturbances are well within the damped region, e.g. $\omega_c \approx 1/2\,\omega_{\text{dist}}$.

The most attractive advantages of our SFD method can be summarized as follows. It is easy to implement into an existing numerical code; it does not require a good initial guess of the solution; steady states can be computed without specific knowledge of the critical bifurcation parameters. To our experience, the SFD method appears to be very robust, and therefore this procedure provides a viable alternative to the classic Newton method.

References

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