Use of the PSE approach for the inviscid jet flow instability and noise prediction.
Comparison with Linearized Euler Equations predictions

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Introduction. The physical origin and the quantitative determination of the noise radiated by a jet flow remains an open scientific challenge. It seems now that several sources may exist, their role being different if the flow is subsonic or supersonic. For supersonic flows, the intrinsic instability of the flow itself is suspected to be the main contributor of the noise (or at least of some part of the radiated noise). In order to check this idea, very detailed comparisons must be done between the linear stability theory (LST) analysis and other approaches. In this paper LST results are compared to Linearized Euler Equations (LEE) computations, the latter being performed by Christophe Bailly in Ecole Centrale de Lyon.

The usual approach applied to the stability analysis of the axisymmetric jet flow is the so-called parallel stability theory. As observed by many authors, this approach leads to severe difficulties when the mean flow becomes stable. It is indeed necessary to extend the physical mean flow field defined as a function of the radial real coordinate r into a non physical flow field defined as a function of the complex coordinate z. Then instead of integrating the stability equations along the real axis, this is done along a deformed path starting and ending at the physical points where the (physical) boundary layer conditions are applied. Furthermore, the deformation is performed such that there are no additional poles between the real axis and the new integration path. Then, according to the residue theorem, the computed eigenvalue along the new path exactly coincides with the one formally defined by the physical stability problem.

However this "by-pass" generates practical difficulties in the computation process of the eigenvalues, it is difficult to have an automatic procedure to define the new complex integration path respecting the mentioned contraints. Furthermore, if the obtained eigenvalue has a physical value, this is not the case of the eigenfunctions which are defined as a function of the complex z along the used integration path, and not of the real radial coordinate. Consequently the expected "detailed comparisons" between LST and other approaches seem to be restricted to the unstable region.

The purpose of the present study is to analyse the capability of the PSE approach with respect to the cited stakes.

The PSE approach has been already used for jet flow instability but as far as the authors know there are no detailed comparisons with other results. The oral presentation will provide first a description of the mean flow, then a short overview of the PSE analysis and will end by some precise comparisons between the two approaches LST and LEE in the region close to the jet (where stability is expected to play an important role) and in the near-field where only acoustic can propagate.

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Mean flow. A hot supersonic mean jet flow is considered. It exits a round nozzle of diameter $D_j = 0.09$ m with a velocity $u_j = 1106$ m/s and a temperature $T_j = 761$ K. The Mach number is then $M_j = 2$ and the ratio between the ambient and the core density is $\rho_\infty/\rho_j = 2.64$. The corresponding mean flow axial velocity can be approximated by different analytical functions depending on the axial position, then the density by a Crocco's relation and finally the radial velocity component (necessary in the PSE approach) is computed from the continuity equation.

PSE approach. Superimposed to the mean flow a fluctuation is sought under the standard PSE form :

$$\tilde{q}(r,\theta,x,t) = \hat{q}(r,x)\exp\left(i\int_{x_0}^{x}\alpha(\xi)\,d\xi\right)\exp\left(i\left(m\theta-\omega t\right)\right) \tag{1}$$

for any fluctuating quantity (three velocity components, density, pressure and temperature). The superposition is introduced into the governing equations written for an inviscid fluid, which are then linearized. Due to the lack of viscous terms, the obtained equations do not need to be "parabolized" : they are or they are not parabolic. Furthermore, a classical mathematical analysis leads to an explicit sufficient condition for these equations to be parabolic :

$$\forall\, x,\ \forall\, r,\quad M_j^2\bar{\rho}(r,x)(\bar{u}_x(r,x))^2 < 1 \tag{2}$$

The system is then closed by a normalization and by appropriate boundary conditions which will be detailed in the presentation.

Some results. As explained before, the stability results are compared to those provided thanks to a LEE (Linearized Euler Equations) code written by Christophe Bailly. In this abstract, are considered only one frequency given by the Strouhal number $St = 0.3$ (calculated with the diameter of the nozzle and the mean axial velocity at the exit) and only one azimuthal number $m = 0$. Both approaches are linear, thus in order to compare them, the amplitudes of the fluctuating pressure in both results have been fixed to be identical at one spatial position in the (x,r) plane : $r/D_j = 4$ and $x/D_j = 10$.

Figure 1 provides a comparison for the streamwise evolution of the fluctuating pressure at four

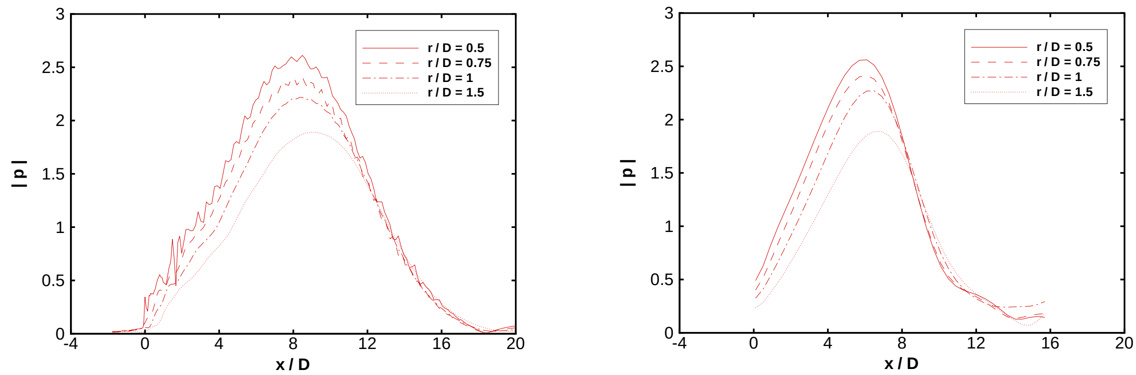

Figure 1: LEE (left figure) and PSE (right figure) streamwise evolution of the pressure amplitude inside the jet for $St = 0.3$ and $m = 0$

different radial positions, which are all inside the mean jet flow. A satisfactory agreement is obtained. This is true in the amplification zone, as already known by previous studies, but this is also true in the damping region : the PSE approach seems to be able to compute the physical eigenfunctions without the tedious procedure necessary with the parallel stability theory (extension of the real axis to a complex integration path).

In the near field, where the mean flow is nearly at rest, the fluctuating pressure is calculated by the same procedure as the one proposed by Ch. Tam in the 80's and usually applied from the parallel theory. Three radial positions are considered in figure 2. Let ut recall that the same constant has been fixed for

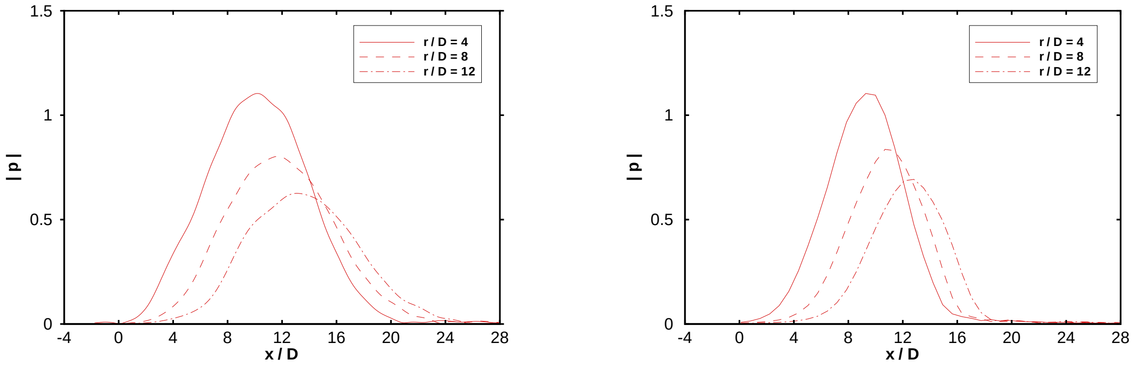

Figure 2: LEE (left figure) and PSE (right figure) streamwise evolution of the acoustic pressure amplitude for $St = 0.3$ and $m = 0$

the two figures, which seems to indicate a good prediction of the radial decay of the pressure fluctuation at the different axial positions.

Finally, figure 3 gives the comparison in the (x, r) plane of the fluctuating pressure amplitude. The

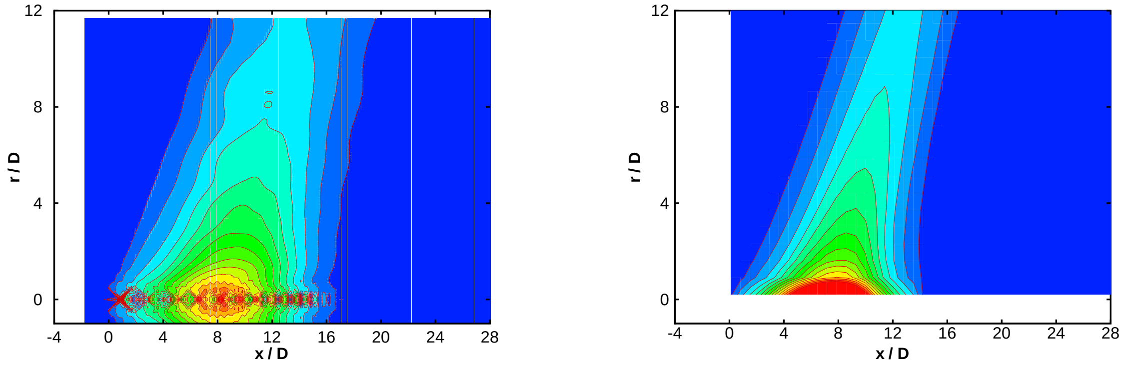

Figure 3: LEE (left figure) and PSE (right figure) spatial evolution of the acoustic pressure amplitude for $St = 0.3$ and $m = 0$

contour levels are the same in both figures.

This study seems to indicate that at least for the considered hot supersonic jet flow, the main contribution of the radiated noise comes from the intrinsic (Kelvin-Helmholtz) instability of the flow.