

Optimal disturbances in the Falkner–Skan–Cooke boundary layer

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It is well known that elongated, streamwise oriented structures develop in boundary layers subjected to sufficiently high levels of free-stream turbulence. These streaks of spanwise alternating low and high streamwise velocity experience algebraic growth even for sub-critical Reynolds numbers and break down to turbulence when they reach a critical amplitude. A number of numerical studies have shown that the growth of non-modal disturbances is maximized when the initial disturbance has the form of streamwise aligned vortices. These optimal disturbances give rise to streamwise streaks with excellent agreement to experimentally measured disturbance profiles.

In the present study a well known adjoint-based procedure is used to calculate optimal disturbances in the Falkner–Skan–Cooke boundary layer (FSC). Corbett & Bottaro ¹ have previously presented the temporal optimal disturbance in this flow, herein the optimization is however carried out within the spatial framework which allows a non-parallel base flow. The motivation for this study comes from the involvement in the EU-project TELFONA, which aims at the development of testing and numerical methodology for laminar wings. Within this project the effect of different turbulence levels in wind-tunnels (relatively high levels) and free-flight conditions (low levels) on transition will be examined. On aircraft wings the sweep angle introduce a cross-flow component which is known to dramatically alter the stability characteristics of the flow. The effects of compressibility, surface curvature and a spatially varying pressure gradient are however equally important. For this reason the base flow has been obtained from solving the boundary layer equations rather than the FSC similarity solution, to allow future studies of these effects.

In swept flows the streaks are known to be more or less aligned with the external streamlines and periodic in the direction parallel to leading edge. A non-orthogonal, curvi-linear coordinate system is therefore introduced, where the first, curved coordinate axis is aligned with the external streamlines and the second and third straight axes are parallel to the leading edge and normal to the surface, respectively. The linearized, disturbed compressible boundary layer equations for this non-orthogonal coordinate system is obtained by Jacobian transformation of the spatial derivatives. By carrying out the disturbance calculations along the streamlines it is possible to assume a homogeneous streamwise wavenumber. In order to verify the code, calculations of optimal disturbances in the 2D Blasius boundary layer has been carried out using a non-orthogonal coordinate system with a curved axis. Excellent agreement with published results was found.

At the present time the investigation of swept boundary layers has just been

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¹Corbett and Bottaro, *J. Fluid Mech.* **435**, pp. 1-23 (2001).

initiated, some initial results will however be presented. As expected, the optimal disturbance in the Falkner–Skan–Cooke boundary layer takes the form of streamwise aligned vortices. These vortices are however tilted in the cross-flow plane, as seen in figure 1.

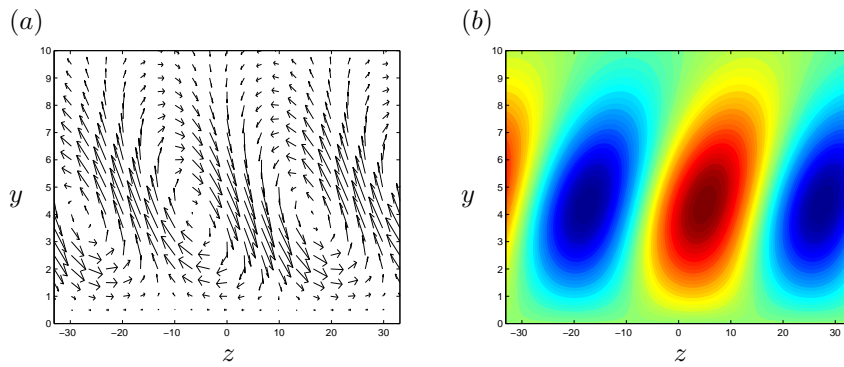


Figure 1: (a) Optimal disturbance in the Falkner–Skan–Cooke boundary layer with favourable pressure gradient and 45° sweep angle. Velocity field in cross-flow plane at inception. (b) Downstream response to the optimal disturbance. Contours of streamwise velocity in the cross-flow plane