

## Control of instabilities in a cavity-driven separated boundary-layer flow

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The two-dimensional incompressible boundary-layer flow along a smooth-edged cavity is considered. The main effect of the cavity is the generation of a recirculation zone with an associated shear layer, as shown in figure 0.1. For the considered aspect ratio of the cavity, unstable global modes appear above a critical inflow Reynolds number based on the boundary layer thickness  $Re_{\delta_0^*} \approx 300$ . This instability is dominated by self-sustained oscillations associated to the familiar Rossiter mechanism Rossiter (1964); small disturbances are amplified by the shear layer through the Kelvin-Helmholtz instability mechanism and generate a pressure wave when impacting on the downstream cavity edge.

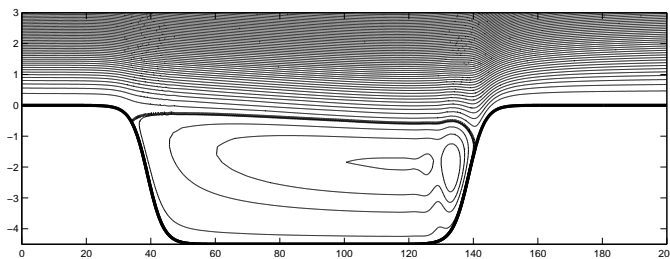


FIGURE 0.1. Streamlines of steady state base flow used for stability analysis. The thick line represents the zero level contour.

We aim at stabilizing the flow in this highly nonparallel configuration using feedback control. When discretizing such a system very large system matrices directly appear. This challenges the construction of the optimal feedback controller, and hence a reduced order model for the flow system is preferable. Usually the reduction is performed by projection on a set of vectors that spans a smaller subspace, the most widely known basis being the set of vectors obtained by balanced truncation (see eg. Skogestad & Postlethwaite (2005)). As an alternative we are using the so called global eigenvectors of the linearized Navier-Stokes as our basis. In some special cases, as for the boundary layer, where one or more of the spatial directions are either homogeneous or slowly varying, different approaches are possible. Expanding in Fourier space leads to a decoupled problem, where optimal gains based on the eigenmodes of Orr-Sommerfeld-Squire equations are computed for each wavenumber (Högberg & Henningson (2002)). In configurations as the present, where the streamwise length scales of the disturbances are comparable to those of the base flow, one needs to consider modes that “live” in the whole domain, the so called Global modes. These modes are computed linearizing the Navier-Stokes system at the steady state  $\mathbf{U}(x, y) = (U(x, y), V(x, y))$ . The disturbance flow field  $\mathbf{u}(x, y, t) = \hat{\mathbf{u}}(x, y) e^{-i\omega t}$  and pressure  $p(x, y, t) = \hat{p}(x, y) e^{-i\omega t}$  is solution of the partial differential system

$$\begin{aligned} -i\omega \hat{\mathbf{u}} &= -(\mathbf{U} \cdot \nabla) \hat{\mathbf{u}} - (\hat{\mathbf{u}} \cdot \nabla) \mathbf{U} - \nabla \hat{p} + \frac{1}{Re} \nabla^2 \hat{\mathbf{u}}, \\ 0 &= \nabla \cdot \hat{\mathbf{u}}. \end{aligned} \quad (0.1)$$

which after discretization is written as

$$-i\omega_l \mathbf{B} \mathbf{q}_l = \mathbf{A} \mathbf{q}_l \quad \text{with adjoint} \quad i\omega_l^* \mathbf{B} \mathbf{q}_l^+ = \mathbf{A}^+ \mathbf{q}_l^+$$

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where the adjoint is defined with the respect to the finite-dimensional inner product. The base flow is interpolated to a Chebyshev-Chebyshev grid. The resulting generalized eigenvalue problem (of size  $n > 50000$ ) is far too large to be solved by standard QZ-algorithms, however Krylov subspace projections together with the Arnoldi algorithm proved to recover the part of the spectrum relevant for our analysis.

The controller is designed for the reduced system, ie. the flow system obtained by projection on the basis of the eigenmodes. The design process involves placements and penalties on actuators and sensors. Sensors measure the shear stress at the downstream lip of the cavity, where the unstable modes are most energetic, and actuators apply upstream, where sensitivity is highest Chomaz (2005). The optimal control loop, in the form of control and estimation feedback gains, is computed through the solution of two Riccati equations.

The controller is updated online, in parallel to the DNS, with a Crank-Nicholson time integration procedure. At each timestep it is forced by measurements from the flow, and outputs a control signal that is fed back to the actuator in the DNS.

Flow stabilization of the evolution due to the worst case initial condition is demonstrated using a reduced model based on projection on a system of only 4 modes.

## Bibliography

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