Exam, SG2214 Fluid Mechanics
11 January 2011, at 09:00-13:00
Examiner: Anders Dahlkild (ad@mech.kth.se)

Copies of Cylindrical and spherical coordinates*, which will be supplied if necessary, can be used for the exam as well as a book of basic math formulas, a calculator and an English dictionary. The point value of each question is given in paranthesis and you need more than 20 points to pass the exam including the points obtained from the homework problems.*

1. (10 p.) Consider the fully developed viscous flow in a tube of radius R mounted with its centre line in the vertical direction. The flow is driven only by gravity, $g\,[\mathrm{m/s^2}]$, and the pressure in the tube may be assumed constant. The fluid is incompressible with density ρ and viscosity μ.

a) Derive an expression, in terms of the given constants above, for the average velocity in the tube $\overline{U} = Q/A$, where $Q\,[\mathrm{m^3/s}]$ is the volume flux in the tube and $A\,[\mathrm{m^2}]$ is the cross section of the tube.

b) Calculate the force on the tube from the fluid per unit length of the tube $F'\,[\mathrm{N/m}]$. Give the direction of this force.

2. (10 p.) Consider the two-dimensional flow field given by

$$u = \frac{a\omega}{kH}\left(1 + \frac{1}{2}k^2(y+H)^2\right)\cos(kx - \omega t)\,,\ v = \frac{a\omega}{kH}k(y+H)\sin(kx - \omega t)\,.$$

The flow field is a linear model for a surface gravity wave on a shallow water layer of depth $H \ll \lambda$, where λ is the wave length. $a \ll H$ is the small amplitude of the wave surface at $y_{\mathrm{surface}} = a\cos(kx - \omega t)$, where ω is the wave frequency and $k = 2\pi/\lambda$ is the wave number. The bottom wall is at $y = -H$.

a) Show that the flow field is irrotational.

b) Show that if $kH = 2\pi H/\lambda \ll 1$ and $a/H \ll 1$ the flow field is approximately incompressible.

c) Calculate the acceleration vector from the given flow field for a material fluid particle at the bottom wall.

3. (10 p.) The complex potential of irrotational 2D flow past a circular cylinder of radius a can be written

$$F(z) = U_\infty\left(z + \frac{a^2}{z}\right) + i\frac{\Gamma}{2\pi}\ln(z/a)\,.$$

a) Find the difference in the fluxes (per unit width) that passes above and below the cylinder surface and below/above the points $(x = 0, y = \pm 2a)$.

b) Find the difference in pressure between the points $(x = 0, y = \pm a)$.

4. (10 p.) The stream function for the flow in a boundary layer along a flat plate can be written

$$\Psi(x,y) = \sqrt{\nu x U_\infty}\, f(\eta)\,,\ \eta = y/\sqrt{\nu x/U_\infty}.$$

a) Use the boundary layer equation to derive Blausius equation for $f(\eta)$. Also give the necessary boundary conditions to solve this equation.

b) Derive an expression for the wall shear stress.

5. (10 p.) Consider the steady state inviscid and incompressible $2D$ stagnation point flow towards an infinite flat plate for which the stream function is given by $\Psi = xy/\tau$, where τ [s] is a given constant. Far away from the plate the temperature is T_∞ and the plate is at a constant temperature $T_w > T_\infty$. The thermal diffusivity of the fluid, $\kappa = k/\rho c_{\mathrm{p}}$, is much larger than the viscous diffusivity $\nu = \mu/\rho$. Therefore, dissipation may be neglected, but heat conduction may not be neglected.

a) Calculate the temperature field, $T(y)$, at steady state using the energy equation.

b) Calculate the heat flux density vector at the plate.

Hints : $\rho c_{\mathrm{p}}\frac{DT}{Dt} = k\nabla^2 T + \Phi$, $\rho, c_{\mathrm{p}},$ and k are constants, $\int_0^\infty e^{-\xi^2}d\xi = \frac{\sqrt{\pi}}{2}$.