Exam, SG2214 Fluid Mechanics
23 October 2010, at 09:00-13:00

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Copies of Cylindrical and spherical coordinates*, which will be supplied if necessary, can be used for the exam as well as a book of basic math formulas, a calculator and an English dictionary. The point value of each question is given in paranthesis and you need more than 20 points to pass the exam including the points obtained from the homework problems.*

1. (10 p.) Use Cartesian tensor notation and write down the complete set of equations governing the flow of a viscous, incompressible fluid (disregarding changes in temperature). Define introduced variables and parameters and make a physical interpretation (description in words) of each of the terms in the equations. In what units (dimensions) are the terms of the equations measured? Define the Reynolds number for a fluid flow example (that you choose yourself) and give a generally valid physical interpretation of the Reynolds number. If the Reynolds number is large, which term in the equations of motion is generally smaller than the other terms. Also, use tensor notation to show that for an incompressible fluid

$$\nabla^2 \mathbf{u} = -\nabla \times \omega,$$

where ω is the curl of the velocity vector.

2. (10 p.) Consider a so called dashpot, a device for damping vibrations in e.g. a machine, see figure 1. A piston can move within a closed cylinder filled with an incompressible viscous oil with viscosity μ. For downward motion of the piston oil must fow through the narrow annular gap between the cylinder and the piston.

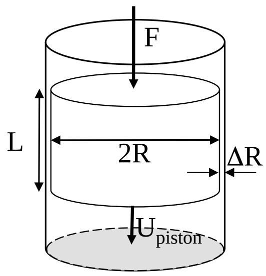

FIGURE 1. Dashpot

a) For given downward velocity of the piston, U_{piston}, and piston radius R determine the volumetric flow rate of oil in the gap.

b) Determine the external force F required as the piston, of length L and radius R, moves at constant speed U_{piston}. Effects of gravity may be neglected and the flow in the narrow gap, ΔR, may be considered a fully developed viscous flow. As $\Delta R << R$ you may assume the cylindrical geometry of the narrow gap to be approximately equivalent to a plane channel with a width equal to the circumferential length of the piston. The Reynolds number of the flow may be assumed very small and details of the flow close to the entrance and exit of the narrow gap may be neglected.

Hints: The pressure levels above and below the piston can be considered as constants.

In fully developed plane channel flow, with one channel wall moving at constant speed and the other channel wall stationary, the velocity profile can be written

$$u(y) = 6\,\overline{U}\ \frac{y}{h}\left(1 - \frac{y}{h}\right) + U_{\text{wall}}\left(1 - \frac{y}{h}\right),$$

where h is the distance between the channel walls and $\overline{U}$ and U_{wall} are constants.

3. (10 p.) The complex potential of an irrotational 2D flow field is given by

$$F(z) = \frac{2}{3}U_0 L\, e^{-i\frac{\pi}{2}}\,(z/L)^{3/2}\,,$$

where U_0 [m/s] and L [m] are real numbers.

a) Find the streamfunction $\Psi(r,\theta)$ in plane polar coordinates and make a rough sketch of the streamlines for $\frac{\pi}{3} \le \theta \le \frac{5\pi}{3}$. In particular you should indicate possible stagnation points in the flow and also streamlines at which $u_\theta = 0$.

b) Calculate the pressure field $p(r,\theta)$ in plane polar coordinates if the pressure at the origin is p_0.

c) Calculate the acceleration vector of a fluid particle. Make sure to indicate the direction of the acceleration vector in the flow field.

4. (10 p.) Consider the 2D flow of an incompressible fluid past a flat plate at $x \ge 0$. The coefficients of viscosity, μ, and heat conduction, k, can be considered as constants. The plate is held at a constant temperature T_0 slightly higher than the temperature of the oncoming flow at temperature T_∞ and velocity U_∞ parallel to the plate.

a) If the Prandtl numer, $Pr = \frac{\mu c_p}{k}$, is a constant of order one (neither large or small), then under what conditions are viscous and thermal effects restricted to a thin boundary layer region close to the plate?

b) The thermal counterpart of the approximate boundary layer equations is given by

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2.$$

State the physical meaning of each term. What simplifications are made as compared to the exact equation? Give the required boundary conditions for T in the present case.

c) The viscous boundary layer over a flat plate has a solution according to Blausius with a stream function of the form

$$\Psi(x,y) = U_\infty\sqrt{\nu x/U_\infty}\; f\left(\eta(x,y)\right),\ \text{where}\ \ \eta(x,y) = \frac{y}{\sqrt{\nu x/U_\infty}}.$$

Make the analogous ansats

$$T(x,y) = T_\infty\;\Theta\left(\eta(x,y)\right)$$

for the temperature field and derive an ordinary differential equation for $\Theta(\eta)$. The velocity field may then be regarded as known from Blausius solution.

d) For Prandtl number $Pr = 1$ one finds a solution for $\Theta(\eta)$ of the form

$$\Theta = -\frac{U_\infty^2}{2c_pT_\infty}\left(f'\right)^2 + Af' + B.$$

Determine the constants A and B and then calculate the heat flux density at the plate in the wall normal direction.

5. (10 p.) Consider the 2D, steady state, incompressible flow past a symmetrical streamlined body at large Reynolds number. Far downstream in the wake at, $x_0 >> L$ (where L is the size of the body), the streamwise velocity is measured and given by $u_w(x_0,y)$. The pressure is found to be approximately equal to the free stream pressure p_∞ (effects of body force/gravity neglected). The measurement region in the vertical direction, $y \in [-y_\infty, y_\infty]$, is large enough that $u_w(x_0,y) \to U_\infty$ as $y \to \pm y_\infty$, where U_∞ is the free stream velocity. Use the momentum theorem and continuity equation on integral form to show that the drag on the body per unit width is given by

$$D'_x = \rho U_\infty^2 \int_{-\infty}^{\infty} \frac{u_w(x_0,y)}{U_\infty}\left(1 - \frac{u_w(x_0,y)}{U_\infty}\right)dy.$$

Hints: Without body force we have

$$\frac{d}{dt}\int_V \rho dV + \oint_S \rho\, u_k n_k\, dS = 0,\ \text{and}\ \ \frac{d}{dt}\int_V \rho u_i dV + \oint_S \rho u_i(u_k n_k)dS = \oint_S \left(-p\delta_{ik} + \tau_{ik}\right) n_k dS.$$