Exam in Fluid Mechanics SG2214

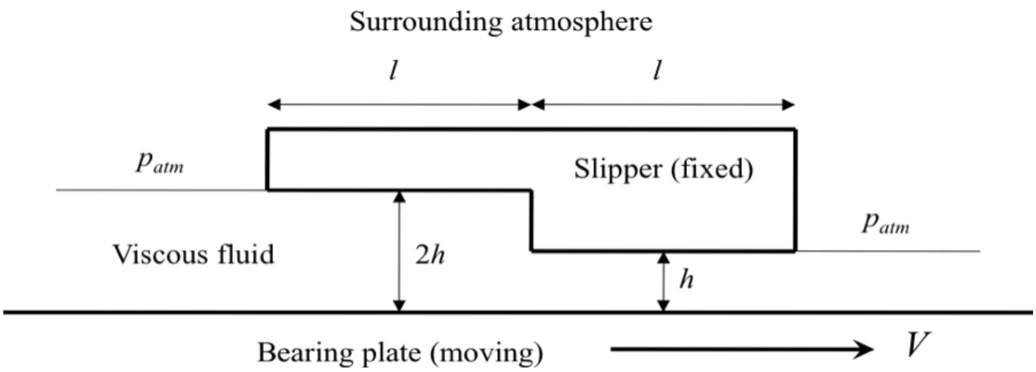
Final exam in course SG2214, January 13, 2010, at 09:00-13:00 in L52.
Examiner: Anders Dahlkild
The point value of each question is given in parenthesis and you need more than 20 points to pass the course including the points obtained from the homework problems.
Copies of Cylindrical and Spherical Coordinates, which will be supplied if necessary, can be used for the exam as well as a book of basic math formulas and a calculator.

1. (10 p.)
a) (5p.) Show that the volumetric flux (per unit width) between two infinite parallel plates, at distance h apart, for fully developed viscous flow is

$$Q = \frac{Vh}{2} - \frac{h^3}{12\mu}\frac{dp}{dx},$$

where one of the plates is moving at velocity V and the other is stationary.

b) (5p.) Calculate the flux (per unit width) in a step bearing if the slipper is stationary and the bearing plate is moving at velocity V according to the figure. Note that the length h is much exaggerated in the figure. As $h << l$ you may assume fully developed parallel flow with a constant pressure gradient in each part of the bearing. (The details of the non-parallel flow near the step and the flow near the in- and outlet need not be considered.)

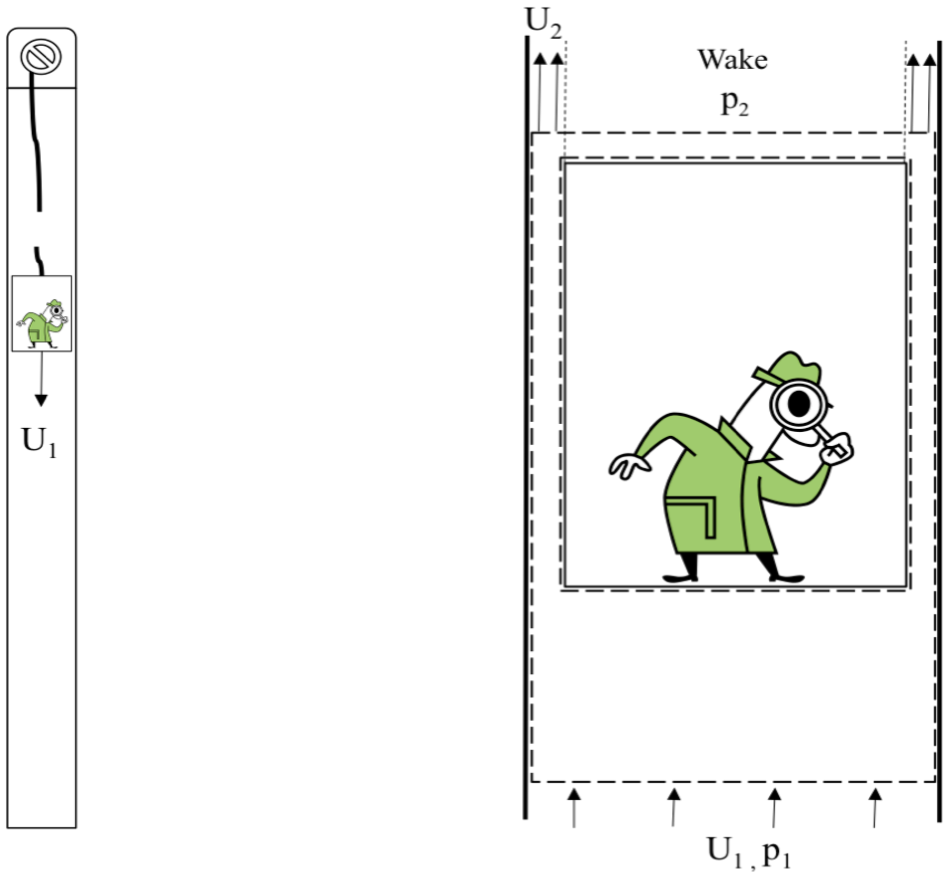


2. (10p.)
Consider the flow field
$u = -\frac{1}{2}\dot{\varepsilon}(1+b)x$, $v = -\frac{1}{2}\dot{\varepsilon}(1-b)y$, $w = \dot{\varepsilon}z$
where b and $\dot{\varepsilon}$ are constants.
a) (1p.) Show that the flow field is incompressible.
b) (3p.) For this flow, describe the deformation of an initially cubic fluid element centred at the origin for
- $b = 0$, $\dot{\varepsilon} > 0$
- $b = 0$, $\dot{\varepsilon} < 0$
- $b = 1$, $\dot{\varepsilon} > 0$

c) (3p.) Calculate the viscous stress (magnitude and direction) on each of the sides of the fluid element for the last case in b).
d) (3p.) Calculate the acceleration field, $\mathbf{a}(x,y,z,t)$, of a material fluid particle in this flow expressed in Eulerian coordinates.

3. (10 p.)
The cable carrying an elevator in a high building is suddenly broken because of an unfortunate circumstance. As the elevator falls due to gravity enclosed air below the elevator has to pass the narrow gap between the elevator and the wall of the elevator shaft. What is the final speed of the elevator (before it hits the ground) if the mass of the elevator is m = 600 kg, the air density is ρ =1.23 kg/m^3, the horizontal cross section of the shaft is A = 1 m^2, the cross section of the gap between elevator and shaft is a = A/10 and gravity g = 9.8 m/s^2? For simplicity, you may neglect effects of viscosity other than the separation of the flow at the edges of the elevator roof. Thus, a wake is formed, which cross-section is A-a just above the roof.

Hint: Put yourself in a coordinate system moving with the elevator facing a constant and uniform flow of air sufficiently far away below the bottom of the elevator. Use the momentum equation in integral form with a control volume according to the figure (dashed) to obtain an expression for the drag force on the elevator.



The momentum equation on integral form, when viscous stresses and body forces on the air are neglected, is given by:

$$\frac{d}{dt}\int_V \rho \mathbf{u} dV + \oiint_S \rho \mathbf{u}(\mathbf{u}\cdot\mathbf{n})dS = \oiint_S (-p)\mathbf{n}dS$$

4. (7 p.)
The Navier-Stokes equations in a rotating frame of reference are given by

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \underbrace{2\Omega \times \mathbf{u}}_{\substack{\text{Coriolis} \\ \text{acceleration}}} + \underbrace{\Omega \times (\Omega \times \mathbf{r})}_{\substack{\text{Centripetal} \\ \text{acceleration}}} = \underbrace{-\frac{1}{\rho}}\nabla p + \mathbf{g} + \nu \nabla^2 \mathbf{u} \ , \quad \nabla \cdot \mathbf{u} = 0$$

The coordinate system is rotating at constant angular velocity Ω, $\mathbf{u}$ is the velocity measured relative to that system and $\mathbf{r}$ is the position vector. Use tensor notation to show that the contribution to the *vorticity* equation from the last two terms on the left hand side and the first term in the right hand side are respectively

$$-2\Omega \cdot \nabla \mathbf{u} \ , \ \mathbf{0} \ , \ \frac{\nabla \rho \times \nabla \mathrm{p}}{\rho^2} .$$

Hint: Note that Ω is a constant vector.

5. (13 p.)
Consider the incompressible flow around a generally shaped body.

a) (2p.) Give the boundary layer equations and state under what conditions they are valid. Make a physical interpretation of each of the terms in the equations. Also state what boundary conditions are necessary.

b) (2p.) Give the characteristics of the "so called" outer flow of a boundary layer and how it may be calculated. What quantities from the outer flow are needed for the boundary layer equations?

c) (2p.) Under what typical conditions may the boundary layer separate from the boundary? What is the limiting condition for the boundary layer profile at separation?

d) (7p.) Derive the Blausius boundary layer equation for a flat plate using a stream function of the form:

$$\Psi(x,y) = U_\infty \delta(x) f(\eta), \quad \text{where} \quad \eta = y/\delta(x), \ \ \delta(x) = \left(\frac{\nu x}{U_\infty} \right)^{1/2} .$$

You should motivate approximations done in the governing equations.