# Questions for preparation for oral exam, Kinetic Theory of Gases

Anders Dahlkild, Lars Söderholm
Mekanik, KTH

December 2011

## Basic physics of a gas

1) When are fluctuations in fluid dynamics quantities like $n, \mathbf{v}, ...$ of importance?

2) What quantitity measures the diluteness of the gas?

3) What is the mean free path $\ell$? Derive an expression for $\ell$.

4) Define the Knudsen number $Kn$. What is typical of $Kn << 1$, $Kn \sim 1$, $Kn >> 1$? How is a typical molecule moving in these three cases?

5) Now assume $Kn << 1$. Give a qualitative picture of how momentum is transported and derive an expression for $\sigma_{xy}$.

## The distribution function $\mathcal{F}$

6) How many molecules are there in a volume element $d^3x$ having velocities in the region $c_i, c_i + dc_i$?

7) What is the value of $\int \mathcal{F} d^3c$ (taken over all velocities)? And the $\int \mathcal{F} d^3x d^3c$ (taken over all of phase space)?

8) How do you obtain $\mathbf{v}$ from $\mathcal{F}$? The stress tensor $\boldsymbol{\sigma}$? The total energy/volume? The heat current $\mathbf{q}$? Show the relation between the total energy/volume and $e$, the total energy/volume in the local rest frame.

9) What does $\mathcal{F}$ look like in equilibrium? Sketch $\mathcal{F}(c_1, 0, 0)$ as a function of $c_1$ for different temperatures. Calculate the stress tensor $\boldsymbol{\sigma}$, the energy/volume in the local rest frame, $e$, and the heat current $\mathbf{q}$ in equilibrium.

## The Boltzmann equation

10) What is a molecular differential cross section $d\sigma/d\Omega$? In particular for hard spheres. What is the molecular interaction potential energy for a hard sphere? Draw a picture. This gives a rough picture of the interaction between molecules. How can it be refined? Draw a picture. How does this potential energy influence the effective size of the molecules for high velocities?

11) Write down the Boltzmann equation. What are the dependent and independent vaiables? Interpret the terms with a plus sign and the terms with a minus sign. Note that the Boltzmann equation can be derived from the Liouville

equation (what is the Liouville equation?) as a first approximation for a dilute gas.

12) Write down the exact conservation laws that follow from the Boltzmann equation.

13) Define the Boltzmann $H$. What equation does it satisfy? When is $dH/dt = 0$ (for what distribution functions) ?

14) Describe the two most important boundary conditions for the Boltzmann equation.

## Small $Kn$

15) When there is a temperature gradient but no flow the expression for $\mathcal{F}$can be derived approximately

$$\mathcal{F}(\mathbf{r},\mathbf{c})\approx\mathcal{F}_0(\mathbf{r},\mathbf{c})(1-\frac{\tau}{2T}\frac{\partial T}{\partial x}c_x(\beta^2c^2-\frac{5}{2})).$$

How is it derived? Sketch the distribution function.

16) The same question when $v_x(y)$ is a shear flow

$$\mathcal{F}\approx\mathcal{F}_0[1-\frac{\tau}{2}\beta^2c'_xc'_y\frac{dv_x}{dy}].$$

17) If we introduce dimensionless variables based on a typical length $L$ where will the Knudsen number - we call it $\varepsilon$here, appear in the Boltzmann equation? Motivate the equations for zero and first order in the Chapman-Enskog expansion.

18) What information about the gas does one need to be able to calculate the viscosity and heat conductivity from solving the first order Chapman-Enskog equation, which is an integral equation?

19) If $Kn$ based on a typical length is small, is it true that the effective $Kn$ is small all the way to the boundary of the gas? What is a Knudsen layer? If the gas is shearing, what is the shape of the tangential velocity close to the boundary? Draw a sketch of the true velocity and also the extrapolation of the velocity outside the Knudsen layer all the way to the wall. What about the temperature close to a wall with given temperature? Motivate that there is a jump in temperature and in tangential velocity at the wall.

20) There is also a phenomen of thermal creep. What is it? Give a physical explanation for this phenomen.

21) What is thermophoresis? Give an example.

22) Write down the boundary conditions for the Navier-Stokes equations when the existence of the Knudsen layer is taken into account. The case where $v_z(x,y)$ and $T(x,y)$ is sufficient.

23) We consider a thin tube with a temperature gradient along the tube. We assume that Navier-Stokes equations with kinetic boundary conditions apply. What is the flow in a closed tube? If instead a thin tube is connected with a wider tube which in turn is connected to the thin tube, what is the flow then? How can this be used to construct a pump without moving parts?

## Large $Kn$

24) Consider Poiseuille flow in a circular tube for free molecular flow. How is the distribution function calculated? What is the velocity profile?

25) Discuss heat transfer between two plates in the free molecular flow limit and compare it to the continuum case. What does the distribution function look like for $Kn >> 1$?

## DSMC and projects

25) What are the main principles of Monte Carlo simulations? When you repeat the simulations you obtain slightly different results? How are they related to fluctuations in the gas (question 1)?

25) Sketch the velocity profiles for Couette flow for small and large Knudsen numbers. Define the Knudsen number in terms of relevant quantities. How can $Kn$ be changed? What does the distribution function look like for $Kn >> 1$?

26) What is the thickness of a shock layer?