Questions for preparing for oral exam in Mathematical Methods of Mechanics

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December 2006

1. Perturbation methods

What is the difference between an asymptotic series and a convergent one?

For a strongly damped linear oscillator what is the small parameter? What perturbation methods can be used?

The same questions for a weakly damped linear oscillator.

What is the meaning of an inner solution and and outer solution. What about the order of the differential equations in the two cases? How many boundary conditions can be satisfied? How to match the two solutions?

How can you obtain a uniformly valid solution?

In fluid dynamics what does the inner solution describe and what the outer solution? What is the small parameter?

Describe the JWKB method

If you have a boundary value problem how can you know where there should be a boundary layer?

2. Nonlinear oscillations

If you have a nonlinear oscillator, what is the problem with naive perturbation methods?

Describe the Poincaré-Lindstedt method and explain in detail why it is better than naive perturbation methods

Suppose it is a Duffing oscillator. What is the small parameter in terms of physical parameters?

What is the difference between the nonlinear result and the linear one?

Write down the equation for the Rayleigh oscillator. Motivate in a qualitative way the existence of a limit cycle.

Now we consider a linear oscillator which is driven by a periodic external force. Why does one in general consider only the solution with the same frequency as the driving force?

Describe the method of multple scales.

Let us for the moment neglect damping. Draw the amplitude of the resulting oscillation as a function of the angular frequency of the driving force. What is

the value of the phase in the different parts of the graph? Give a physical explanation of what happens close to the eigenfrequency.

Draw the corresponding curve for the Duffing oscillator. Compare with the linear case. In particular, there are regions for which there is more than one solution with the same period as the driving force. Are they all possible in practice? How can one find out about their stability?

3. Linear sound waves

What is a soundwave? Derive the wave equation for a fluid.

Derive the general solution for the 1+1 dimensional wave equation

What is the general form of spherical waves?

What is the relation between the wave equation and the Helmholtz equation?

What is a Green function? What does it look like for the Helmholtz equation?

Derive the Green function for the wave equation in 3+1 dimensions

Write down the Green function in the 2+1 dimensional case. Which is the essential difference between the 3+1 and 2+1 cases?

What is the use of a Green function?

How can you use a Green function to solve problems in a half space?

Why does $\triangle$ acting on $1/r$ give a δ-function?

Give an overview of how to obtain the flow field around a supersonic slender body. Why is it essential that the body is slender?

What does the flow look like? Where is the Mach cone? What is the variation with distance from the body?

4. Dispersive waves

What is the difference between phase velocity and group velocity? Derive the expression for the group velocity

Derive the equations for water waves. Sketch the phase velocity and group velocity as a function of wave number.

What is the expression for the frequency for deep water?

Are short waves on water gravity waves or is there some other mechanism as well?

What is the condition for the applicability of the method of stationary phase? How is the group velocity important in this connection?

5. Kinematic waves and nonliner soundwaves

How can the wave equation be simplified if one considers waves propagating in one dimension only?

Under what assumption can one derive the kinematic wave equation from the equation of continuity?

Give the implicit solution of the kinematic wave equation and sketch the solution

Show that sound waves propagating in one direction obey the kinematic wave equation

An important parameter for nonlinear sound waves is the nonlinearity parameter β. How is it obtained?

Sound waves have constant entropy. What about shock waves?

What is a shock wave? Why do shocks develop?

What is the speed of a shock wave?

Is a shock infinitely thin?

What does the solution of the kinematic wave equation look like when a shock has appeared?

In spite of the strange appearance of the solution it can be used to construct the real solution with the shock. How?

Show one of the methods of equal area

Tell about the other method.

6. Home assignments

You should also be prepared to tell about your home assignments