Mekanik-KTH
2000:09

KUNGL. TEKNISKA HÖGSKOLAN

Notes on kinetic-equation approach of fluid-dynamic equations

Yoshio Sone

Stockholm, 2000

Royal Institute of Technology
Department of Mechanics

Mekanik-KTH
2000:09

Notes on kinetic-equation approach of fluid-dynamic equations

Yoshio Sone

Notes on kinetic-equation approach of fluid-dynamic equations

Yoshio Sone
230-133 Iwakura-Nagatani-cho Sakyo-ku Kyoto 606-0026 Japan

Abstract

Kinetic-equation approach of numerical study of fluid-dynamic equations is discussed. A method constructing a kinetic equation replacing the fluid-dynamic-type equation exactly is proposed. On the basis of this method, the validity or improvement of heuristic numerical kinetic-equation systems is discussed.

1 Introduction

There is a recent trend to make use of kinetic-type equation in obtaining numerical solutions of the Euler or Navier-Stokes set of equations (e.g., [1], [2], [3]). The way of computation is different among the authors, but they noticed the simple linear form of the differential term of the kinetic equation and try to avoid some difficulties in numerical analysis of the Euler or Navier-Stokes set (e.g., instability problems in shock wave computations, the pressure term in the incompressible Navier-Stokes set, preference of quick computation of complex problems to detailed precise one). They made some success in various problems.

One of the direction of this trend is by Vasseur's. He uses the simplest part of kinetic equation, that is, the Boltzmann equation without collision term but, instead, modifies the velocity distribution function at each time step. For this scheme he developed the mathematical theory, though it is limited to the case where the ratio of the specific heats is equal to three. Kun Xu's approach is also in this direction, although he uses the BKW equation and introduces some technique to treat the discontinuity (shock wave or contact discontinuity) in the flow field. He investigated many problems by his method.[4]

In the present brief notes, we propose a simple way to construct a kinetic system of equation and initial condition from a set of partial differential equation of a conservation form (e.g., Euler and Navier-Stokes sets) in such a way that some moments of the solution of the kinetic system satisfy the set of partial differential equations exactly. On the basis of this kinetic system, we discuss the numerical scheme of Vasseur type as an example. This type of discussion has wide application to construction of efficient schemes and evaluation of their validity.

2 Exact kinetic-equation approach

Let x_i, ξ_i, and t be independent variables, and $f(x_i, \xi_i, t)$ be a dependent variable (or the velocity distribution function). The (macroscopic) variables ρ_r $(r = 0, 1, \cdots, 4)$ are defined as follows:

$$\rho_r = \int \gamma_r f \mathrm{d}\xi_1 \mathrm{d}\xi_2 \mathrm{d}\xi_2, \tag{1}$$

where

$$\gamma_0 = 1,\ \gamma_i = \xi_i,\ \gamma_4 = \xi_i^2, \tag{2}$$

and the fluxes H_i^r of the macroscopic variables are defined as follows:

$$H_i^r = \int \xi_i \gamma_r f \mathrm{d}\xi_1 \mathrm{d}\xi_2 \mathrm{d}\xi_2. \tag{3}$$

Take a velocity distribution function f_c of Chapman-Enskog type where the space and time variables enter only through the macroscopic variables or their space variables:

$$f(x_i, \xi_i, t) = f_c(\rho_r, \nabla\rho_r, \xi_i), \tag{4}$$

where ∇ is the representative of a collection of the derivatives $\partial^s/\partial x_{i_1} \cdots \partial x_{i_s}$. Naturally,

$$\rho_r = \int \gamma_r f_c(\rho_r, \nabla\rho_r, \xi_i) \mathrm{d}\xi_1 \mathrm{d}\xi_2 \mathrm{d}\xi_2.$$

The fluxes H_i^r corresponding to the velocity distribution function $f_c(\rho_r, \nabla\rho_r, \xi_i)$ of Chapman-Enskog type is denoted by $\hat{H}_i^r$, that is,

$$\hat{H}_i^r(\rho_r, \nabla\rho_r) = \int \xi_i \gamma_r f_c(\rho_r, \nabla\rho_r, \xi_i) \mathrm{d}\xi_1 \mathrm{d}\xi_2 \mathrm{d}\xi_2. \tag{5}$$

Now, choosing a function $f_c(\rho_r, \nabla\rho_r, \xi_i)$ of Chapman-Enskog type, we discuss a solution of the following functional equation:

$$\begin{aligned}
\frac{\partial f}{\partial t} + \xi_i \frac{\partial f}{\partial x_i} = \xi_i & \left(\frac{\partial f_c}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial f_c}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i} \right) \\
& - \frac{\partial f_c}{\partial \rho_q} \left(\frac{\partial \hat{H}_i^q}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial \hat{H}_i^q}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i} \right) \\
& - \frac{\partial f_c}{\partial \nabla \rho_q} \nabla \left(\frac{\partial \hat{H}_i^q}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial \hat{H}_i^q}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i} \right),
\end{aligned} \tag{6}$$

with the initial condition:

$$f(x_i, \xi_i, t) = f_c(\rho_r^0, \nabla\rho_r^0, \xi_i) \quad \text{at } t = t_0. \tag{7}$$

This equation has the same form of conservation equation as the equation without the right-hand side terms:

$$\frac{\partial \rho_r}{\partial t} + \frac{\partial H_i^r(\rho_r, \nabla \rho_r)}{\partial x_i} = 0. \tag{8}$$

This is derived from the following manipulation of the right-hand side:

$$\begin{aligned}
&\int \gamma_r \xi_i \left(\frac{\partial f_c}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial f_c}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i} \right) \mathrm{d}\xi_1 \mathrm{d}\xi_2 \mathrm{d}\xi_2 \\
&= \frac{\partial \hat{H}_i^r}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial \hat{H}_i^r}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i},
\end{aligned}$$

and

$$\begin{aligned}
&\int \gamma_r \frac{\partial f_c}{\partial \rho_q} \left(\frac{\partial \hat{H}_i^q}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial \hat{H}_i^q}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i} \right) \mathrm{d}\xi_1 \mathrm{d}\xi_2 \mathrm{d}\xi_2 \\
&= \left(\frac{\partial \hat{H}_i^q}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial \hat{H}_i^q}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i} \right) \frac{\partial}{\partial \rho_q} \int \gamma_r f_c \mathrm{d}\xi_1 \mathrm{d}\xi_2 \mathrm{d}\xi_2 \\
&= \left(\frac{\partial \hat{H}_i^q}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial \hat{H}_i^q}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i} \right) \frac{\partial \rho_r}{\partial \rho_q} \\
&= \frac{\partial \hat{H}_i^r}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial \hat{H}_i^r}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i},
\end{aligned}$$

and

$$\begin{aligned}
&\int \gamma_r \frac{\partial f_c}{\partial \nabla \rho_q} \nabla \left(\frac{\partial \hat{H}_i^q}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial \hat{H}_i^q}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i} \right) \mathrm{d}\xi_1 \mathrm{d}\xi_2 \mathrm{d}\xi_2 \\
&= \nabla \left(\frac{\partial \hat{H}_i^q}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial \hat{H}_i^q}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i} \right) \frac{\partial}{\partial \nabla \rho_q} \int \gamma_r f_c \mathrm{d}\xi_1 \mathrm{d}\xi_2 \mathrm{d}\xi_2 \\
&= \nabla \left(\frac{\partial \hat{H}_i^q}{\partial \rho_s} \frac{\partial \rho_s}{\partial x_i} + \frac{\partial \hat{H}_i^q}{\partial \nabla \rho_s} \frac{\partial \nabla \rho_s}{\partial x_i} \right) \frac{\partial \rho_r}{\partial \nabla \rho_q} \\
&= 0.
\end{aligned}$$

Proposition: The solution of the initial value problem (6) with (7) is assumed to be unique. Then the set of the macroscopic variables ρ_r calculated from the solution $f(x_i, \xi_i, t)$ is the solution of the initial value problem of the partial differential equations

$$\frac{\partial \rho_r}{\partial t} + \frac{\partial \hat{H}_i^r(\rho_r, \nabla \rho_r)}{\partial x_i} = 0, \tag{9}$$

with the initial condition

$$\rho_r = \rho_r^0 \quad \text{at } t = t_0. \tag{10}$$

Proof: Take a function

$$f(x_i,\, \xi_i,\, t) = f_c(\rho_r^c(x_i,t), \nabla\rho_r^c(x_i,t), \xi_i), \tag{11}$$

where $\rho_r^c(x_i,t)$ is the solution of the following partial differential equations:

$$\frac{\partial \rho_r^c}{\partial t} + \frac{\partial \hat{H}_i^r(\rho_r^c, \nabla\rho_r^c)}{\partial x_i} = 0, \tag{12}$$

with the initial condition

$$\rho_r^c = \rho_r^0 \quad \text{at } t = t_0. \tag{13}$$

Then

$$\rho_r(x_i,t) = \rho_r^c(x_i,t),$$

$$\begin{aligned}
\frac{\partial f}{\partial t} &= \frac{\partial f_c}{\partial \rho_r^c}\frac{\partial \rho_r^c}{\partial t} + \frac{\partial f_c}{\partial \nabla\rho_r^c}\frac{\partial \nabla\rho_r^c}{\partial t} \\
&= -\frac{\partial f_c}{\partial \rho_r^c}\frac{\partial \hat{H}_i^r(\rho_r^c, \nabla\rho_r^c)}{\partial x_i} - \frac{\partial f_c}{\partial \nabla\rho_r^c}\nabla\left(\frac{\partial \hat{H}_i^r(\rho_r^c, \nabla\rho_r^c)}{\partial x_i}\right),
\end{aligned}$$

and

$$\xi_i\frac{\partial f}{\partial x_i} = \xi_i\frac{\partial f_c(\rho_r^c, \nabla\rho_r^c, \xi_i)}{\partial \rho_r^c}\frac{\partial \rho_r^c}{\partial x_i} + \xi_i\frac{\partial f_c(\rho_r^c, \nabla\rho_r^c, \xi_i)}{\partial \nabla\rho_r^c}\frac{\partial \nabla\rho_r^c}{\partial x_i}.$$

Thus,

$$\begin{aligned}
\frac{\partial f}{\partial t} + \xi_i\frac{\partial f}{\partial x_i} &= \xi_i\frac{\partial f_c(\rho_r^c, \nabla\rho_r^c, \xi_i)}{\partial \rho_r^c}\frac{\partial \rho_r^c}{\partial x_i} + \xi_i\frac{\partial f_c(\rho_r^c, \nabla\rho_r^c, \xi_i)}{\partial \nabla\rho_r^c}\frac{\partial \nabla\rho_r^c}{\partial x_i} \\
&\quad - \frac{\partial f_c(\rho_r^c, \nabla\rho_r^c, \xi_i)}{\partial \rho_r^c}\frac{\partial \hat{H}_i^r(\rho_r^c, \nabla\rho_r^c)}{\partial x_i} \\
&\quad - \frac{\partial f_c(\rho_r^c, \nabla\rho_r^c, \xi_i)}{\partial \nabla\rho_r^c}\nabla\left(\frac{\partial \hat{H}_i^r(\rho_r^c, \nabla\rho_r^c)}{\partial x_i}\right) \\
&= \xi_i\frac{\partial f_c(\rho_r, \nabla\rho_r, \xi_i)}{\partial \rho_r}\frac{\partial \rho_r}{\partial x_i} + \xi_i\frac{\partial f_c(\rho_r, \nabla\rho_r, \xi_i)}{\partial \nabla\rho_r}\frac{\partial \nabla\rho_r}{\partial x_i} \\
&\quad - \frac{\partial f_c(\rho_r, \nabla\rho_r, \xi_i)}{\partial \rho_r}\frac{\partial \hat{H}_i^r(\rho_r, \nabla\rho_r)}{\partial x_i} \\
&\quad - \frac{\partial f_c(\rho_r, \nabla\rho_r, \xi_i)}{\partial \nabla\rho_r}\nabla\left(\frac{\partial \hat{H}_i^r(\rho_r, \nabla\rho_r)}{\partial x_i}\right).
\end{aligned}$$

Thus the function f defined by Eq. (11) with the subsidiary conditions (12) and (13) is a solution of the initial-value problem (6) with (7) From the uniqueness,

the set of the macroscopic variables ρ_r obtained the solution f of the initial-value problem is the solution of the partial differential equations (9) with the initial condition (10). ■

As the first example, take the Maxwellian distribution

$$f_c = \frac{\rho}{(2\pi RT)^{3/2}} \exp\left(-\frac{(\xi_i - u_i)^2}{2RT}\right), \tag{14}$$

as a velocity distribution function of Chapman-Enskog type, where the macroscopic variables ρ_r are related to the parameters ρ, u_i, and T in the Maxwellian as

$$\rho_0 = \rho, \quad \rho_i = \rho u_i, \quad \rho_4 = \rho(3RT + u_i^2). \tag{15}$$

and the fluxes $\hat{H}_i^r$ for the Maxwellian are given by

$$\hat{H}_i^0 = \rho u_i, \quad \hat{H}_i^j = \rho(u_i u_j + RT\delta_{ij}), \quad \hat{H}_i^4 = \rho u_i(5RT + u_j^2). \tag{16}$$

The macroscopic variables ρ_r (or ρ, u_i, and T) derived from the solution of this kinetic equation satisfy the Euler set of equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0, \tag{17a}$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} + \frac{\partial R\rho T}{\partial x_i} = 0, \tag{17b}$$

$$\frac{\partial \rho(3RT + u_j^2)}{\partial t} + \frac{\partial \rho u_i(5RT + u_j^2)}{\partial x_i} = 0. \tag{17c}$$

As the second example, take the distribution function:

$$f_c = \frac{\rho}{(2\pi RT)^{3/2}} \exp\left(-\tilde{\zeta}^2\right) \left[1 - \frac{a(T,\rho)}{(2RT)^{1/2}} \frac{\partial T}{\partial x_i} \tilde{\zeta}_i (\tilde{\zeta}^2 - \frac{5}{2}) \right.$$
$$\left. - b(T,\rho) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \left(\tilde{\zeta}_i \tilde{\zeta}_j - \frac{\tilde{\zeta}^2}{3}\delta_{ij}\right)\right], \tag{18}$$

where

$$\tilde{\zeta}_i = \frac{\xi_i - u_i}{(2RT)^{1/2}}, \tag{19}$$

and $a(T,\rho)$ and $b(T,\rho)$ are functions of T and ρ, which can be chosen according to our convenience. The macroscopic variables ρ_r are related to the parameters ρ, u_i, and T in the distribution function (18) as

$$\rho_0 = \rho, \quad \rho_i = \rho u_i, \quad \rho_4 = \rho(3RT + u_j^2). \tag{20}$$

and the fluxes $\hat{H}_i^r$ are given by

$$\hat{H}_i^0 = \rho u_i, \tag{21a}$$

$$\hat{H}_i^j = \rho(u_i u_j + RT\delta_{ij}) - b(T,\rho)(R\rho T)\left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right], \tag{21b}$$

$$\hat{H}_i^4 = \rho u_i(5RT + u_j^2) - b(T,\rho)(R\rho T)u_j\left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right] - \frac{5a(T,\rho)(R\rho T)}{2}\frac{\partial T}{\partial x_i}. \tag{21c}$$

The macroscopic variables ρ_r (or ρ, u_i, and T) derived from the solution of this kinetic equation satisfy the Navier-Stokes set of equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0, \tag{22a}$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} + \frac{\partial R\rho T}{\partial x_i} = \frac{\partial}{\partial x_j}\left\{b(T,\rho)(R\rho T)\left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right]\right\}, \tag{22b}$$

$$\frac{\partial \rho(3RT + u_j^2)}{\partial t} + \frac{\partial \rho u_i(5RT + u_j^2)}{\partial x_i} = \frac{\partial}{\partial x_j}\left\{b(T,\rho)(R\rho T)u_i\left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right]\right\} + \frac{\partial}{\partial x_j}\left[\frac{5a(T,\rho)(R\rho T)}{2}\frac{\partial T}{\partial x_i}\right]. \tag{22c}$$

These are the Navier-Stokes set of equations with viscosity $b(T,\rho)(R\rho T)$ and thermal conductivity $5a(T,\rho)(R\rho T)/2$. Thus choosing $a(T,\rho)$ and $b(T,\rho)$ properly, we obtain the Navier-Stokes set with arbitrary viscosity and thermal conductivity. It should be noted that the form (18) of f_c is not necessary to be taken to be the same distribution function of the Chapman-Enskog expansion that gives the Navier-Stokes set with the same viscosity and thermal conductivity.

In the examples, the function f_c is somewhere resembled with the Chapman-Enskog expansion, but it is not required to be so. We can choose freely according to the request that the function gives the required set of equations. Equation (14) or (18) is just an example giving the Euler set or Navier-Stokes set.

3 Discussion on numerical systems

In the previous section, a kinetic-equation system that describes fluid-dynamic equations exactly is presented. In the present section, we discuss the validity of heuristic numerical kinetic-equation systems. The kinetic-equation approach which is interested in here is the following one or its variant: The solution of an initial-value problem of the Euler set of equations is discussed by the following

kinetic equation in place of solving the Euler set directly. That is, the solution of Eq. (23) in a continuous series of intervals $(t_0, t_1], (t_1, t_2], (t_2, t_3], \cdots, \cdots$

$$\frac{\partial f}{\partial t} + \xi_i \frac{\partial f}{\partial x_i} = 0, \tag{23}$$

under the initial condition of each interval

$$f = \frac{\rho_{(m)}}{(2\pi R T_{(m)})^{3/2}} \exp\left(-\frac{(\xi_i - u_{i(m)})^2}{2RT_{(m)}}\right) \quad \text{at } t = t_m, \tag{24}$$

where $\rho_{(0)}$, $u_{i(0)}$, and $T_{(0)}$ are the real initial value at $t = t_0$ for the Euler set, and $\rho_{(m)}$, $u_{i(m)}$, and $T_{(m)}$ $(m \neq 0)$ are ρ, u_i, and T calculated from f at $t = t_m$ obtained as the solution of Eq. (23) in the interval $(t_{m-1}, t_m]$. The ρ, u_i, and T of the solution f at arbitrary time is taken the solution of the corresponding initial-value problem of the Euler set.

The above system being compared with the exact system corresponding to Eq. (14) in the previous system, the difference of the kinetic equations is that on their right-hand side, which is of the order of unity. With the same initial condition, the error of the velocity distribution function f is order of $t_{(m+1)} - t_{(m)}$ in the interval $(t_{(m)}, t_{(m-1)}]$. That is, $|f - f_c| = O(t_{(m+1} - t_{(m)})$. Our interest is the error of the variables ρ, u_i, and T. These variables or their equivalents ρ_r satisfy the conservation equations

$$\frac{\partial \rho_r}{\partial t} + \frac{\partial H_i^r}{\partial x_i} = 0, \tag{25}$$

and the variables ρ^c, u_i^c, and T^c (or ρ_r^c) satisfy

$$\frac{\partial \rho_r^c}{\partial t} + \frac{\partial \hat{H}_i^r(\rho_r^c, \nabla \rho_r^c)}{\partial x_i} = 0. \tag{26}$$

The difference between fluxes H_i^r and $\hat{H}_i^r(\rho_r^c, \nabla \rho_r^c)$ are bounded by the difference between f and f^c by definition [see Eqs. (3) and (5)], that is,

$$|H_i^r - \hat{H}_i^r| = O(|f - f^c|) = O(t_{(m+1)} - t_{(m)}). \tag{27}$$

Thus from Eqs. (25) and (26),

$$|\rho_r - \rho_r^c| = O((t_{(m+1)} - t_{(m)})^2). \tag{28}$$

The the initial distribution function $f_{(m+1)}$ at $t = t_{(m+1)}$:

$$f_{(m+1)} = \frac{\rho_{(m+1)}}{(2\pi R T_{(m+1)})^{3/2}} \exp\left(-\frac{(\xi_i - u_{i(m+1)})^2}{2RT_{(m+1)}}\right) \tag{29}$$

differs from f^c at $t = t_{(m+1)}$ only by $O((t_{(m+1)} - t_{(m)})^2)$. Continuing the discussion to the next interval, we obtain the same error estimate with the subscripts $(m+1)$ and (m) shifted by unity. Thus, the errors of the distribution function

f and the macroscopic variables ρ_r at a finite time t from f^c and ρ_r^c are of the order of $\max|t_{(m+1)} - t_{(m)}|$. That is, by modifying the distribution function f to the corresponding Maxwellian $f_{(m)}$ at every end $t_{(m)}$ of time interval, we can limit the error within the order of the maximum length of divided time intervals. Thus, the kinetic system described at the beginning of this section (say, system of free molecular type) is an appropriate one to obtain the solution of the Euler set of equations (the first order system). By choosing an appropriate term on the right-hand side of Eq. (23), we can construct a higher-order system.

By a similar discussion, we find that the solution of the Navier-Stokes set of equations is obtained by the kinetic system of free molecular type with modification of the initial condition at the beginning of every interval to the function f^c given by Eq. (18). For the first-order system, the kinetic system of free molecular type is sufficient. It often happen that the viscous and thermal conduction terms [a and b in Eq. (18)] are small corrections and that we would like to obtain the solution correct up to that order. Then, for a reasonable computation avoiding too small time step, we should choose the right hand side of Eq. (6) with f_c given by Eq. (14) as the right-hand side of Eq. (23).

The direct simulation Monte-Carlo (DSMC method) developed by Bird[5] is widely used in engineering problems of a rarefied gas flow. The hybrid-method that combines the DSMC and fluid-dynamic (Euler or Navier-Stokes) schemes is developed by several authors (e.g., [6], [7], and references there) to save the computing time of the DSMC computation in the fluid-dynamic region, where the effective Knudsen number (the local mean free path divided by the local characteristic length scale of variation of the variables) is small. The main difference among various works is the technique of transition to one region to the other between two quite different schemes. Vasseur's method of kinetic-equation approach reminds us the direct simulation DSMC method in the way: Free molecular computation corresponds to free motion of the particles in DSMC, and modification of the distribution function to Maxwellian in each time step corresponds to changing the velocities of some particles by statistical collision simulation. If the second process of the DSMC computation to modify particle velocities by the statistical process is replaced by modifying the distribution of the particle to the local Maxwellian (24) or the distribution e.g., of the type by Eq. (18), the DSMC computation can be carried out more efficiently in the fluid-dynamic region. In combination of the original DSMC method and the DSMC version of Vesseur's approach, the transition is done naturally and can be another candidate to time saving in addition of the existing domain splitting method.

Acknowledgment

This note is a part of the talk by the author at the seminar at Department of Mechanics KTH (Stockhlom) on 4th September 2000. The collaboration between KTH and Kyoto is continued since the author gave the Enskog Lecture at KTH in 1997. The author thanks Professor Söderholm for his arrangement

and support for the seminar.

References

[1] A. Vasseur, "Convergence of a semi-discrete kinetic scheme for the system of isentropic gas dynamics with $\gamma = 3$," Preprints, Département de Mathématiques et Informatique, Ecole Normale Supérieure (LMENS-98-3)

[2] Kun Xu, "Gas-kinetic schemes for unsteady compressible flow simulations" von Karman Institute for Fluid Dynamics (Belgium) report 1998-03 (1998).

[3] T. Inamuro, M. Yoshino, and F. Ogino, "Accuracy of the lattice Boltzmann method for small Knudsen number with finite Reynolds number," Phys. Fluids **9**, 3535-3542 (1997).

[4] Kun Xu, "Gas-kinetic schemes for fluid simulation," Invited lecture at the 1st conference on computational fluid dynamics (Kyoto, Japan, 2000).

[5] G. A. Bird, *Molecular Gas Dynamics and the Direct Simulation of Gas Flows*, (Oxford University Press, Oxford, 1994).

[6] S. Tiwari and A. Klar, "An adaptive domain decomposition procedure for Boltzmann and Euler equation," J. Comp. Appl. Math. **90**, 223-237 (1998).

[7] A. Garcia, J. Bell, Wm. Y. Crutchfield, and B. Adler, "Adaptive Mesh and Algorithm Refinement using Direct Simulation Monte Carlo," J. Comp. Phys. **154**, 134-155 (1999).

Royal Institute of Technology
Department of Mechanics
S-100 44 Stockholm, Sweden