Figure 1:

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Continuum Mechanics
Home assignment number 5 2008
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Rigid motion

In a rigid motion with one point $\mathbf{X}_0$ fixed

$$\mathbf{x} = \mathbf{X}_0 + \mathbf{R}(t)(\mathbf{X} - \mathbf{X}_0).$$

Prove that

$$\mathbf{v} = \mathbf{W}\mathbf{r},$$

where

$$\mathbf{W} = \dot{\mathbf{R}}\mathbf{R}^T$$

and $\mathbf{r}$ is the vector from $\mathbf{X}_0$

$$\mathbf{r} = \mathbf{x} - \mathbf{X}_0.$$

Show that $\mathbf{W}$ is antisymmetric, so that

$$\mathbf{W} = \boldsymbol{\omega}\times,$$

where $\boldsymbol{\omega}$ is the angular velocity vector. Or for any vector $\mathbf{c}$

$$\mathbf{W}\mathbf{c} = \boldsymbol{\omega} \times \mathbf{c}.$$

Also show that the acceleration $\mathbf{a}$ is

$$\mathbf{a} = (\dot{\mathbf{W}} + \mathbf{W}^2)\mathbf{r}$$

or

$$\mathbf{a} = \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

Newton's bucket

A bucket of radius R is filled with water with density ρ up to height H. It is then set into rotation with angular velocity ω around the vertical axis. Eventually, the water in the bucket will carry out the same rigid motion as the bucket. Find the shape of the surface of the water, including the height of its lowest and highest points. The acceleration of gravity is g. The air pressure is p_0.

Use the Euler equations to find the distribution of pressure in the water. Then apply the boundary condition to find the shape of the water surface.

Can you alternatively interpret your result as a solution of the equlibrium equations in the rotating frame?