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A light wave which is monocromatic, i.e. of the form

$$e^{-i\omega t}\phi(x)$$

obeys the Helmholtz equation

$$\frac{d^2\phi}{dx^2} + k^2\phi = 0.$$

Here, the wave number is

$$k = \frac{\omega}{c}.$$

We are considering an inhomogeneous medium, so that the wave speed c is given as

$$c = \frac{c_0}{n(x)}.$$

Here, $n(x)$ is the refractive index. So we can write the Helmholtz equation

$$\frac{d^2\phi}{dx^2} + k_0^2(n(x))^2\phi = 0.$$

We now assume that the refractive index is given by ($a > 0$)

$$n(x) = 1 + a\exp(-(x/L)^2).$$

For $x << -L$

$$\phi = A\exp(ik_0x).$$

Find the wave for $x >> L$. What are the conditions for the problem to be possible to solve using the WKB method? Assume that these are satisfied.

What is the amplitude of the wave for $x >> L$? What is the extra phase compared with the undisturbed wave?

If the initial wave is divided up into two waves, one of which passes through the inhomogeneity but the other one not, one can see to it that the phase difference between them is precisely π or $-\pi$. If the two waves are then made to superpose again, they will interfere destructively and the total intensity is reduced to zero.