Figure 1:

Institutionen för mekanik
Lars Söderholm, tel 7152
lars.soderholm@mech.kth.se

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A particle of small mass M with a hole drilled through it can slide along the Y-axis. The particle is connected with a damper, giving it a damping force $-BdY/dT$. The particle is also connected with one spring, with one end fixed at the point $X = l_0$ on the X-axis and with an identical spring fixed at $X = -l_0$. The spring constant is k. When the particle is at the origin, the spring is in its relaxed state. Show that to lowest order in Y the component of the spring force in the direction of motion of the particle is not linear in Y as is usually the case for a spring, but cubic, that it is given by $-(k/(l_0^2))Y^3$. So now we have a cubic force rather than a linear one.

$$M\frac{d^2Y}{dT^2} + B\frac{dY}{dT} + \frac{k}{l_0^2}Y^3 = 0$$

Assume that the particle starts at $Y(0) = a$. It is natural to use a as a scale of Y. Write

$$Y = ay$$

This means that $y(0) = 1$.

$$M\frac{d^2y}{dT^2} + B\frac{dy}{dT} + Ky^3 = 0$$

So we have three constants in the equation

$$M, B, K = k(\frac{a}{l_0})^2.$$

From them we can form one dimensionless combination

$$\varepsilon = \frac{MK}{B^2}.$$

Strong damping means that this is a small quantity. When the inertia term can be neglected, after the transient, as $y \sim 1$, the characteristic time is $\sim T_1 = B/K$. The particle is assumed to arrive at $Y = a/3$ at time $T = B/K$.

Introduce a dimensionless time $t = T/T_1$ and find T_1 such that the coefficients of the spring term and the damping term are the same. Show that the equation is

$$\varepsilon \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \frac{\mathrm{d}y}{\mathrm{d}t} + y^3 = 0.$$

What is the timescale of the transient? In terms of t and in terms of the time T?

This means that we have to find a solution to the differential equation, which satisfies

$$y(0) = 1; y(1) = 1/3.$$

Find this solution. You need to calculate it to lowest (or zeroth) order in ε only. But it should be valid for all t.