Figure 1:

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We are considering an acoustic wave. It satisfies the equation

$$v_t + \beta v v_x = 0.$$

At $t = 0$ v consists of two triangular pulses, v is given by

$$\begin{aligned} 2v_0\frac{x}{a},\ 0 &< x < a \\ 2v_0\frac{(2a-x)}{a},\ a &< x < 2a \\ v_0\frac{x-2a}{a},\ 2a &< x < 3a \\ v_0\frac{4a-x}{a},\ 3a &< x < 4a \end{aligned}$$

For all other values of x v vanishes.

a) Introduce dimensionless variables x^*, t^*, v^* such that a and v_0 disappear from the problem and write down the equation and intial conditions in the new variables.

b) When will the larger pulse develop a shock? When will the smaller pulse develop a shock? Give the shape of the wave when there are two shocks. Draw a picture of it. At a later time, the stronger shock will catch up with the weaker shock. When will that be?

Hint: when the two shocks merge, the value of v immediately to the right of the left shock has to be the same as the value of v immediately to the left of the right shock.