Homework 1A 5C1212, due 2/1, 2006

1. An airplane is flying with a trajectory given by $x_i = r_i(t)$. It measures the temperature field $T(t, x_1, x_2, x_3)$ in the surrounding air with a thermometer fixed to the wing. Find an expression for the rate of change of the temperature. Compare this expression with that of the material derivative.
2. Consider the fluid flow between two infinite parallel plates, where the upper plate is moving at a constant velocity V, and the lower plate is fixed. Find a steady solution to the incompressible Navier-Stokes for this flow case, i.e. the so called plane Couette flow.
3. Consider the inviscid flow with constant velocity U over a semi infinite flat plate. Let the incomming flow have a temperature T_0 while the plate is kept at the higher temperature T_w. As was shown in the lecture notes, for incompressible flow the temperature field around the plate obeys the equation
$$\frac{DT}{Dt} = \kappa \nabla^2 T$$
where κ is the thermal conductivity of the fluid. Assume that the Peclet number, $Pe = Re\, Pr$, is large so that a thin *thermal* boundary layer develops on the plate. Derive the appropriate boundary layer approximation of the temperature equation in the limit of large Peclet number and estimate the thermal boundary layer thickness.
4. Mean heat equation. The heat equation for the instantaneous turbulent flow is
$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \kappa \frac{\partial^2 T}{\partial x_j \partial x_j}$$
Derive the heat equation governing the mean temperature $\bar{T}$.