Computational Fluid Dynamics (5C1212), 5p.

Lecturers:

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Literature:

Copies from J.C. Tannehill, D.A. Anderson, R.H. Pletcher, *Computational Fluid Mechanics and Heat Transfer*, Taylor & Francis, 1997, (TAP)

Copies from Randall J LeVeque, *Fininte Volume Method for Hyperbolic Problems*, Cambridge University Press, 2002 (LeV)

D. Henningson, *Lecture notes on Computational Fluid Dynamics*

K. Gustavsson, *Lecture notes on Basic Numerics*

Grading:

Test total of 50p., homeworks (required) $3 \times 5 = 15$p.

Total points > 30 (3), > 40 (4), > 50 (5).

Web links

http://www.mech.kth.se/

Course plan

1. Tue 18/1 8-10, E51. Fluid dynamics I: LB

   Introduction and outline of the course.

   Derivation of the governing equations.

2. Wed 19/1 15-17, E51. Basic numerics I: KG

   Mathematical behavior of hyperbolic, parabolic and elliptic equations. Well-posedness.
   KG Lecture notes

3. Thu 20/1 8-10, E35. Fluid dynamics II: LB

   Derivation of the governing equations , cont.

4. Fri 21/1 15-17 L21. Basic numerics II: KG

   Discretization by finite differences. Analysis of discretized equations; order of accuracy, convergence and stability (von Neumann analysis).
   TAP: 3.1-3.3.5, 3.6, LeV: 8-8.2, 8.3.3

5. Tue 25/1 8-10, E51. Fluid dynamics III: LB

   Derivation of the governing equations , cont.

6. Wed 26/1 15-17, Q36. Basic numerics III: KG

   Numerical methods for model equations related to different levels of approximation of Navier Stokes equation: linear wave equation, Burgers equation, convection-diffusion equation. First and second order numerical methods such as upwind, Lax-Friedrichs, Lax-Wendroff, MacCormack, etc. Modified equation - dissipation and dispersion.
   TAP: 4.1-4.1.8, LeV: 8.6

7. Thu 27/1 8-10, E35. Fluid dynamics IV: LB

   Dimensionless form, fluid phenomena, simplified equations

8. Fri 28/1 15-17, E52. Basic numerics IV: KG

   Numerical methods for model equations, cont.

9. Mon 31/1 13-15, Q35. Compressible flow I: KG

   Introduction to compressible flow. Euler equations, conservative/non-conservative form. Some thermodynamics.
   LeV: 14

   Scalar conservations laws: Conservation, weak solutions, non-uniqueness, entropy conditions. Shock formation, Rankine-Hugoniot relations.
   LeV: 11

10. Wed 2/2 15-17, E52. Compressible flow II: KG

    Numerical methods for scalar conservation laws. Properties of the numerical scheme such as CFL-condition, conservation and TVD. First order methods. System of conservations laws
    LeV: 4.1-4.9, 3

11. Fri 4/2 15-17, E52. Compressible flow III: KG

    Numerical methods for Euler equations: MacCormack and artificial viscosity for non-linear systems. Numerical/physical boundary conditions. Shock tube problem.
    LeV: 3, 14

12. Tue 8/2 08-10, E51 . Compressible flow IV: KG

    High resolution schemes for conservations laws.
    LeV: 6

13. Wed 9/2 15-17, D34. Compressible flow V: KG

    Numerical methods for Euler equations. Boundary conditions, Riemann invariants. Compressible flow in 2D.
    TAP: 6.3-6.7, LeV: 14

14. Fri 11/2 15-17, E35. Compressible flow VI: KG

    Numerical methods for Euler equations, cont. Grids, algebraic mesh generation by transfinite interpolation. Flow around an airfoil.

15. Tue 15/2 08-10, E3. Finite volume and finite difference methods I: DH

    Laplace equation on arbitrary grids, equivalence with finite-differences, linear systems: Gauss-Seidel as smothers for multi-grid.

16. Wed 16/2 15-17, D34. Finite volume and finite difference methods II: DH

    Introduction to incompressible flow. Properties of the equations, role of the pressure: artificial compressibility and projection on divergence free space, Navier-Stokes in integral form.

17. Fri 18/2 15-17, E35. Finite volume and finite difference methods III: DH

    Staggered grid/volume formulation + BC. Unsteady equations: projection and MAC method, discrete Poisson pressure eq.

18. Tue 22/2 8-10, Q34. Finite volume and finite difference methods IV: DH

    Time step restrictions. Steady equations: distributive iteration and SIMPLE methods.

19. Thu 24/2 8-10, E35. Finite elements I: DH

    An advection–diffusion problem. Variational form of the equation, weak solutions, essential and natural boundary condition. Finite-element approximations, stability and accuracy, the algebraic problem, matrix assembly.

20. Fri 25/2 15-17, E31. Finite elements II: DH

    Navier–Stokes equations. Mixed variational form, Galerkin and FE approximations, the algebraic problem. Stability, the LBB condition, mass conservation.

21. Tue 1/3 13-15, E31. Finite elements III: DH

    Navier–Stokes equations. Mixed variational form, Galerkin and FE approximations, the algebraic problem. Stability, the LBB condition, mass conservation.

22. Wed 2/3 15-17, Q33. Finite elements IV: DH

    Navier–Stokes equations. Mixed variational form, Galerkin and FE approximations, the algebraic problem. Stability, the LBB condition, mass conservation.

23. Sat 2/4 9-13, E31. EXAM.

Homeworks

- Homework 1A, due 31/1: Homework on NS equations.
- Homework 1B, due 31/1: Numerical methods for model equations. Different schemes. Dispersion, diffusion. Stability analysis.
- Homework 2, due 15/2: Numerical methods for non-linear conservation laws. Shock-tube.
- Homework 3, due 22/2: Running compressible Euler code; flow over a bump.
- Homework 4, due 1/3: FV homework + numerical project.
- Homework 5, due 8/3: FEM homework.